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Selling a vote

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Abstract

A voting function is a rule that determines the outcome of an election: taking the voters' votes as input, a voting function selects the winning candidate from the set of candidates receiving some vote. A voting function is immune to vote selling when, given that neither voter *i* nor voter *j* votes for the winning candidate, a change ceteris paribus in *i*'s vote cannot make the candidate for which *j* votes the winner. It is shown that voting functions immune to vote selling have either a dictator (a voter who always determines the winning candidate) or a dictated candidate (a candidate who becomes the winner by just receiving some vote).

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1. Introduction

A voting function is a rule that determines the outcome of an election: taking the voters' votes as input, a voting function selects the winning candidate from the set of candidates receiving some vote. As Feldman (1980, p. 210) remarks, the legitimacy of the elected candidate depends on whether voters have voted sincerely, with a voter voting sincerely if he votes for one of his most preferred candidates. This observation makes the task of identifying voting functions that are immune to nonsincere voting relevant.

To fulfil this task, it is necessary to specify what voters care about. A first approach consists of presuming that voters exclusively care about candidates. In this setting, and when voters can rank the candidates from the most preferred to the most disliked, the Gibbard (1973)- Satterthwaite (1975) theorem shows the impossibility of having a non-

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dictatorial voting function in which each candidate could win in at least one election and where, by voting nonsincerely, no voter can induce the voting function to select a candidate that the voter prefers to the candidate elected when he votes sincerely.

In a second approach, voters do not only care about candidates but define preferences over candidates and some other elements not directly related to the election itself but owned by the voters. Such elements would be relevant insofar as they could be transferred from one voter to another as a way to induce the latter to sell his vote to the former. For instance, voters may be assumed to define preferences over both candidates and amounts of money, so that they can tell whether they prefer having a certain candidate elected or having a certain amount of money and another candidate elected.

To keep the formal model as simple as possible, the voters' preferences will not be specified and the emphasis will be placed on (trying to remove) the *possibility* that a voter could have the opportunity to sell his vote to another voter. This opportunity is assumed to occur when the winning candidate is not the one for which a certain voter i votes but, by changing his vote, another voter j can make i's candidate the winner. It could of course be that the amount of money i is willing to pay j is not enough to compensate j for all the troubles caused by changing his vote or that j does not accept any amount of money to change his vote. One possible justification of focusing on trying to remove that possibility is precisely not having to care about such considerations.

Remark 2.3 shows that, if voters whose candidate is elected can sell their votes, then no voting function is immune to vote selling, namely, every voting function grants such voters the possibility of selling their votes. This justifies restricting attention to the case in which only voters whose candidate is not elected can sell their votes. After all, it is reasonable to expect that the price such voters ask for selling their votes is lower than the price asked by voters whose candidate is elected. For this reason, a voter eager to buy some vote will contact first voters whose candidate is not the winning candidate. If most voting functions were immune to vote selling in this restricted case, the message for voters willing to buy a vote would be that they should expect to pay a high price.

The main result of the paper (Proposition 3.1) shows that only two seemingly undesirable types of voting function remove the possibility of such a restricted form of vote selling. Specifically, if all possible ways of voting for candidates are allowed and there are at least three voters and three candidates, then there exist just two ways of removing the opportunity for vote selling: by having a dictator or a dictated candidate. Consequently, to prevent the possibility that dissatisfied voters have the power to alter the outcome of the election in a way satisfactory to another dissatisfied voter, either a single voter must always determine the elected candidate or a certain candidate must be elected whenever some voter votes for that candidate.

2. Definitions

The n members of a finite set N of voters face the problem of choosing a candidate from a given finite set A of m candidates on the basis of information reported by the voters themselves, where the information a voter reports is an element of a certain message set M. Adapting Moulin (1994, p. 1101), let a game form be a pair (M, f), with $f: M^n \to A$ being