Mathematical Social Sciences 74 (2015) 41–59

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/econbase

Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: An update

William Thomson

University of Rochester, Department of Economics, Harkness Hall, 14627 Rochester, NY, United States

HIGHLIGHTS

- Paper surveys work on the adjudication of conflicting claims since Thomson (2003).
- The bulk of the survey is on the axiomatic approach, as most of the recent work has been along axiomatic lines.
- It discusses extensions of the base model and applications.
- It outlines directions for further research.

ARTICLE INFO

Article history:
Received 19 August 2013
Received in revised form
31 August 2014
Accepted 27 September 2014
Available online 5 January 2015

ABSTRACT

A group of agents have claims on a resource but there is not enough of it to honor all of the claims. How should it be divided? A group of agents decide to undertake a public project that they can jointly afford. How much should each of them contribute? This essay is an update of Thomson (2003), a survey of the literature devoted to the study of such problems.

© 2015 Published by Elsevier B.V.

1. Introduction

A group of agents have claims on a resource, but there is not enough of it to honor all of the claims. How should it be divided? Although societies have had to deal with situations of this type since time immemorial, their formal study began in earnest with O'Neill (1982). O'Neill describes a number of fascinating historical examples dating to antiquity and medieval times, together with the resolutions proposed for them at that time. He suggests a mathematical representation of the problem of how to adjudicate conflicting claims, develops several methods to handle it, axiomatic and game-theoretic, and applies these methods to derive a number of rules. A survey covering the literature that this seminal paper generated is Thomson (2003), hereafter referred to as T2003.[1]

The model has other interpretations. It covers in particular the problem faced by a group of agents undertaking a public project and having to decide how much each of them should contribute (hence the reference to taxation in our title), but for simplicity, we will use language that pertains to the adjudication of conflicting claims.

It is remarkable how quickly the literature developed. An important reason is undoubtedly that researchers could take advantage of the conceptual apparatus and of the techniques developed in other branches of the axiomatics of resource allocation. The study of how to adjudicate conflicting claims is quite rewarding, and it has a unique place in this program. Indeed, the model is one for which many interesting rules can easily be defined. Also, several central requirements that are often too strong to be met in other contexts are satisfied by many rules here.

The field has kept growing. Since the publication of T2003, a number of gaps were closed. The implications of various axiom systems are much better understood today and new axiomatic perspectives have been explored. Particularly significant are advances in the study of consistency and of the distributional implications of rules. Also, O'Neill's model has been enriched in a variety of ways. Thus, an update appeared useful. We take up in turn each of the topics covered in T2003, and in each case, we describe what we have learned in the last twelve years. We also indicate new trends.

Section 2 introduces the model and the rules that have come up in its analysis. Section 3 reports on the progress made on the axiomatic front. Section 4 focuses on its game-theoretic modeling, both cooperative and strategic. Section 5 discusses experimental work. Section 6 presents the various ways in which O'Neill's model has been adapted to accommodate broader classes of environments. We have also updated the references of a number of papers that we had discussed in T2003 but were not yet published.

E-mail address: wth2@mail.rochester.edu.

[1] Pedagogical expositions are by Malkevitch (0000, 2008, 2009).

http://dx.doi.org/10.1016/j.mathsocsci.2014.09.002
0165-4896/© 2015 Published by Elsevier B.V.

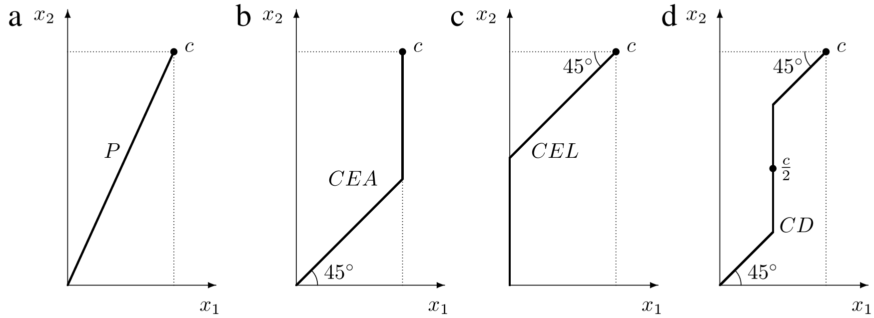

Fig. 1. Paths of awards of four central rules. Typical paths for $N \equiv \{1, 2\}$ and $c \in \mathbb{R}^N_+$. (a) Proportional rule. (b) Constrained equal awards rule. (c) Constrained equal losses rule. (d) Concede-and-divide.

2. The base model of adjudication of conflicting claims and an inventory of rules

The notation and most of the language we use are as in T2003, except for a few terms, which we have replaced by ones that we feel are more informative. We keep the overlap with T2003 to the minimum necessary for a self-contained exposition. Readers familiar with the literature can skip this section, devoted to basic definitions.

2.1. The model

In order to distinguish the model introduced by O'Neill (1982) from the enriched models that have been formulated recently (Section 6), we refer to it as the **base** model. It is as follows. Let $N \equiv \{1, \ldots, n\}$ be a set of **claimants**. Each claimant $i \in N$ has a **claim** $c_i \in \mathbb{R}_+$ on an **endowment** $E \in \mathbb{R}_+$. The endowment is insufficient to honor all of the claims. Altogether, a claims problem, or simply a **problem**, is a pair $(c, E) \in \mathbb{R}^N_+ \times \mathbb{R}_+$ such that $\sum c_i \geq E$. Let $\mathcal{C}^N$ be the class of all problems. An **awards vector of** (c, E) is a vector $x \in \mathbb{R}^N$ satisfying **non-negativity** (no claimant should be asked to pay: $x \geqq 0$), **claims boundedness** (no claimant should be awarded more than his claim: $x \leqq c$), and **balance** (the sum of the awards should be equal to the endowment: $\sum x_i = E$). A **rule** is a function that associates with each $(c, E) \in \mathcal{C}^N$ a unique awards vector of (c, E). Our generic notation for a rule is the letter S. The **path of awards of a rule for the claims vector c** is the locus of the choice it makes as the endowment ranges from 0 to $\sum c_i$.

We will also consider the generalization of the model obtained by letting the population of claimants vary. Then, there is an infinite set of potential claimants, indexed by the natural numbers, $\mathbb{N}$. In each problem, only finitely many of them are present however. Let $\mathcal{N}$ be the family of all finite subsets of $\mathbb{N}$. Still using the notation $\mathcal{C}^N$ for the class of problems with claimant set N, a rule is now defined on $\bigcup_{N \in \mathcal{N}} \mathcal{C}^N$: it associates with each $N \in \mathcal{N}$ and each $(c, E) \in \mathcal{C}^N$, an awards vector of (c, E).

Given $a, b \in \mathbb{R}^N$, $\boldsymbol{seg}[\boldsymbol{a}, \boldsymbol{b}]$ denotes the segment connecting these two points.

2.2. Rules

All of the following rules will come up at some point. We define them for a fixed N. Let $(c, E) \in \mathcal{C}^N$. It will simplify some definitions to assume that no two claims are equal. The adjustments necessary to cover possible equality of claims are straightforward. The first four definitions are illustrated in Fig. 1.

For the **proportional rule**, ***P***, (Aristotle and Thompson, 1985), for each $i \in N$, claimant i's award is λc_i, λ being chosen, as in the next two definitions, so that awards add up to E.

For the **constrained equal awards rule**, ***CEA***, (Maimonides, 12th Century, 2000) claimant i's award is $\min\{c_i, \lambda\}$. An algorithmic definition will be useful, keeping $c \in \mathbb{R}^N_+$ fixed and letting the endowment grow from 0 to $\sum c_j$. At first, equal division takes place until each claimant receives an amount equal to the smallest claim. The smallest claimant drops out, and we divide the next increments in the endowment equally among the others until each of them receives an amount equal to the second smallest claim. The second smallest claimant drops out, and so on.

For the **constrained equal losses rule**, ***CEL***, (Maimonides, 12th Century, 2000) claimant i's award is $\max\{c_i - \lambda, 0\}$. A symmetric algorithm to that underlying the constrained equal awards rule can be defined. Let $c \in \mathbb{R}^N$. This time we let the endowment decrease from $\sum c_j$ – then each claimant is fully compensated – to 0. At first, we impose equal losses on all claimants until their common loss is equal to the smallest claim. The smallest claimant receives 0 then, and he drops out. As the endowment continues to decrease, we maintain equality of losses for the others until their common loss is equal to the second smallest claim. The second smallest claimant drops out, and so on.

Concede-and-divide (Aumann and Maschler, 1985) is the two-claimant rule that first assigns to each claimant the difference between the endowment and the other agent's claim (or 0 if this difference is negative), and divides the remainder equally.

The **Talmud rule**, ***T***, (Aumann and Maschler, 1985) is a hybrid of the constrained equal awards and constrained equal losses rules: it selects $CEA(\frac{c}{2}, E)$ if $E \leq \frac{\sum c_i}{2}$, and $\frac{c}{2} + CEL(\frac{c}{2}, E - \frac{\sum c_i}{2})$ otherwise. An algorithm producing it is obtained by applying in succession the algorithms generating the constrained equal awards and constrained equal losses rules, but using as switchpoints the half-claims instead of the claims themselves. For $|N| = 2$, the Talmud rule coincides with concede-and-divide. The **reverse Talmud rule** (Chun et al., 2001) is derived from this definition by exchanging the roles played by the constrained equal awards and constrained equal losses rules. For $|N| = 2$, we obtain **reverse concede-and-divide**.[2]

Letting once again the endowment grow from 0 to $\sum c_i$, the **constrained egalitarian rule** (Chun et al., 2001) selects $CEA(\frac{c}{2}, E)$ until the endowment reaches $\frac{\sum c_i}{2}$. We assign the next increments to the smallest claimant until he receives the maximum of his claim and half of the second smallest claim. We divide the next increments equally between the two smallest claimants until the smallest claimant receives his claim, in which case the second smallest claimant receives each additional unit until he receives the maximum of his claim and half of the third smallest claim, or they reach half of the third smallest claim; and so on.

Piniles' rule, ***Pin***, (Piniles, 1861) results from a "double" application of the constrained equal awards rule using, as for the Talmud rule, the half-claims instead of the claims themselves: it selects $CEA(\frac{c}{2}, E)$ if $E \leq \frac{\sum c_i}{2}$, and $\frac{c}{2} + CEA(\frac{c}{2}, E - \frac{\sum c_i}{2})$ otherwise.

2 Chun (2005) defines another variant of the Talmud rule.

Download English Version:

https://daneshyari.com/en/article/972552

Download Persian Version:

https://daneshyari.com/article/972552

Daneshyari.com