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Two kinds of Pareto improvements of the economic system: An input–output analysis using the nonnegative matrices theory

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HIGHLIGHTS

- Define the Pareto improvement of final output rate and input multiplier.
- Define the Pareto improvement of value-added rate and output multiplier.
- Give a sufficient condition for Pareto improvement of final output rate and input multiplier.
- Give a sufficient condition for Pareto improvement of value-added rate and output multiplier.
- Define the partitional row and column indicator sets for a reducible nonnegative matrix.

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ABSTRACT

This paper clearly reveals the economic meanings of final output rate, input multiplier, value-added rate, and output multiplier. Applying the output adjustment model and the price adjustment model as well as nonnegative matrices theory we find that if the matrix of intermediate output (or input) coefficients has at least one non-final (or non-initial) class then (i) an adjustment of output system enables the final output rates of whole or part sectors corresponding to the non-final classes to rise, the input multipliers of whole or part sectors corresponding to the non-final classes to decrease, and all other sectoral final output rates and input multipliers to be fixed, which is one kind of Pareto improvement of the economic system; (ii) the adaptation of price system enables the value-added rates of whole or part sectors corresponding to the non-initial classes to rise, the output multipliers of whole or part sectors corresponding to the non-initial classes to decrease, and all other sectoral value-added rates and output multipliers to be unchanged, which is another kind of Pareto improvement of the economic system. We respectively give a sufficient condition for the two kinds of Pareto improvements. A numerical example verifies these results.

1. Introduction

Brown et al. (1973) and Brown and Licari (1977) introduced the "price adjustment model" in order to research the socialist price mechanism. Dietzenbacher (1997) analyzed the relationship between Ghosh's "supply-driven" input–output model and Leontief's traditional "demand-driven" input–output model, where the price adjustment model (see Dietzenbacher, 1997, formula (9)) and the output adjustment model *in value terms* (see Dietzenbacher, 1997, formula (12)) were included, though he did not use such terminologies. Zeng (2008) established the *physical* output adjustment model, discussed output adjustment model and price adjustment model as well as their basic natures, and revealed the effects of changes in outputs on final output rates and input multipliers and the effects of changes in prices on value-added rates and output multipliers. Zeng (2010) discovered the equivalent conditions that all sectoral final output rates equal their respective sectoral value-added rates and that all sectoral input multipliers equal their respective sectoral output multipliers.

From Zeng (2008, 2010), the accommodation of output system can alter two kinds of indicators: final output rates and input multipliers; also the adaptation of price system can alter two kinds of indicators: value-added rates and output multipliers. However, what are the comprehensive economic meanings of the four kinds of indicators? Only the input multiplier and the output multiplier are partly explained via Miller and Blair (1985, Chapter 9), the value-added rate is preliminarily interpreted by Zeng (2008), and yet the final output rate is not clarified.

Via Zeng (2008, 2010), an adjustment of output system enables some sectoral final output rates to rise and all others to be constant, and/or some sectoral input multipliers to decrease and all others

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to be fixed. Nevertheless, how is the output system adjusted? Zeng (2008, 2010) does not present the corresponding method though the proof of Zeng (2008, Theorem 1) gives us an inkling. Besides, which sectoral final output rates can be risen? Which sectoral input multipliers can be decreased? Zeng (2008, 2010) does not discuss these problems. Similarly, the adaptation of price system enables some sectoral value-added rates to rise and all others to be unchanged, and/or some sectoral output multipliers to decrease and all others to be constant. However, Zeng (2008, 2010) does not offer the matching method of adapting the price system. Moreover, which sectoral value-added rates can be risen? Which sectoral output multipliers can be decreased? Zeng (2008, 2010) does not expound these problems.

Based on Zeng (2008, 2010) this paper will solve the aforesaid problems, which will be very useful to improve the four kinds of indicators of the economic system and to develop the close relationship between input–output economic theory and nonnegative matrices theory, which are just the main purposes of this paper.

First we shall completely give the economic meanings of the four kinds of indicators. Also we shall definitely indicate that by adjusting the output system some sectoral final output rates are risen and all others are fixed, and some sectoral input multipliers are decreased and all others are unchanged, this is one kind of Pareto improvement of the economic system, which can be called the *Pareto improvement of final output rate and input multiplier*, and that by adapting the price system some sectoral value-added rates are risen and all others are constant, and some sectoral output multipliers are decreased and all others are fixed, this is another kind of Pareto improvement of the economic system, which can be called the *Pareto improvement of value-added rate and output multiplier*. The two kinds of Pareto improvements are dual. Next we shall present the concrete method of adjusting the output system, which is really a sufficient condition for the Pareto improvement of final output rate and input multiplier. At the same time, the sectoral types of final output rates to be risen and of input multipliers to be decreased are roughly divided by the non-final class(es) of the matrix of intermediate output coefficients. Dually, we shall offer the concrete method of adapting the price system, which is actually a sufficient condition for the Pareto improvement of value-added rate and output multiplier. At the same time, the sectoral types of value-added rates to be risen and of output multipliers to be decreased are roughly divided by the non-initial class(es) of the matrix of intermediate input coefficients. Finally we shall verify the above theory and approaches via a numerical example.

The paper is organized as follows.

In Section 2, the economic meanings of the four kinds of indicators are revealed. It is indicated that the higher the final output rate is, the smaller the input multiplier is, which is more beneficial to the economic system, and that the higher the value-added rate is, the smaller the output multiplier is, which is more profitable to the economic system. By Definition 1 we establish four new concepts of the partitional row (or column) indicator sets corresponding to non-final (or non-initial) class(es) and to final (or initial) class(es) for a Frobenius normal form of a reducible nonnegative matrix. Definition 1 and Theorem 1 are the indispensable mathematical foundations of this paper.

In Section 3, a new concept of the Pareto improvement of final output rate and input multiplier is founded. Dually, a new concept of the Pareto improvement of value-added rate and output multiplier is presented.

In Section 4, the concrete approach to the Pareto improvement of final output rate and input multiplier is expounded. Dually, the concrete approach to the Pareto improvement of value-added rate and output multiplier is elucidated. Theorems 2 and 3 are the cores.

In Section 5, a numerical example illustrates and verifies the main results, where the two kinds of Pareto improvements of the economic system are simulated.

2. Preliminary analysis and concepts

2.1. Notations and terminologies

For the general notations and terminologies, let logical notations $\exists, \forall, \wedge, \vee$, and $\Leftrightarrow$ represent existence, arbitrariness, conjunction, disjunction, and equivalence, respectively. Via $\Gamma \Rightarrow \Omega$, we denote that Γ implies Ω. The set notations $\in, \cup$, and $\cap$ stand for belong, union of sets, and intersection of sets, respectively. The empty set is indicated by ϕ. Let 0 be zero, a zero vector or zero matrix. A vector or matrix $M > 0$ means that M is *semipositive*, i.e. every entry of M is nonnegative, and $M \neq 0$. A vector or matrix $M \gg 0$ implies that M is *positive*, viz. every element of M is positive. A vector or matrix M is called *strictly semipositive*, if $M > 0$ and at least one entry is zero. Evidently, M is semipositive if and only if M is positive or strictly semipositive. By M^t we indicate the transpose of vector or matrix M. Let $\rho(M)$ represent the spectral radius of a matrix M. Let $\hat{H} = \text{diag}(H) = \text{diag}(h_1, h_2, \ldots, h_n)$ be the diagonal matrix with $h_1, h_2, \ldots, h_n$ as its main diagonal entries, where H is a column or row vector. The unit matrix is represented via I. The unit column vector $E = (1, 1, \ldots, 1)^t$.

Furthermore, let $q \gg 0$ be a column vector of physical gross outputs; $p \gg 0$, a row vector of prices; $X = \hat{p}q \gg 0$, a column vector of gross output values, and thus $X^t \gg 0$, a row vector of gross input values; $x = E^tX = X^tE > 0$, the sum of all sectoral gross output values or gross input values; $F = (f_i)_{n\times 1} > 0$, a column vector of monetary final demands (or final output values); $Y = \hat{X}^{-1}F > 0$, a column vector of final output rates; $V = (v_j)_{1\times n} > 0$, a row vector of values-added (or primary input values); $R = V\hat{X}^{-1} > 0$, a row vector of value-added rates; $Z > 0$, a monetary transaction matrix for the intermediate products; $ZE > 0$, a column vector of intermediate output values; $E^tZ > 0$, a row vector of intermediate input values; $w = E^tZE > 0$, the sum of all sectoral intermediate output values or intermediate input values; $\bar{A} > 0$, a matrix of physical intermediate input coefficients; $\vec{A} = \hat{X}^{-1}Z = (\vec{a}_{ij})_{n\times n} > 0$, a matrix of intermediate output coefficients; $\vec{A}E > 0$, a column vector of intermediate output rates; $\vec{B} = (I - \vec{A})^{-1} = (\vec{b}_{ij})_{n\times n} > 0$, a Ghosh inverse matrix; $G = \vec{B}E = (g_i)_{n\times 1} \gg 0$, a column vector of input multipliers; $A = Z\hat{X}^{-1} = (a_{ij})_{n\times n} > 0$, a matrix of monetary intermediate input coefficients; $E^tA > 0$, a row vector of intermediate input rates; $B = (I - A)^{-1} = (b_{ij})_{n\times n} > 0$, a monetary Leontief inverse matrix; $D = E^tB = (d_j)_{1\times n} \gg 0$, a row vector of output multipliers; where $\rho(\bar{A}) = \rho(\vec{A}) = \rho(A) < 1$, $\vec{A} = \hat{q}^{-1}\bar{A}\hat{q} = \hat{X}^{-1}A\hat{X}$, $\vec{B} = \hat{X}^{-1}B\hat{X}$, and $A = \hat{p}\bar{A}\hat{p}^{-1}$ (see also Miller and Blair, 1985, Chapter 9). Let $Q \gg 0$ be a column vector of output adjustment coefficients; $P \gg 0$, a row vector of price adjustment coefficients; the superscript # correspond to the new output system; the superscript * correspond to the new price system; where $\vec{A}^{\#} = \hat{Q}^{-1}\vec{A}\hat{Q}$, $\vec{B}^{\#} = \hat{Q}^{-1}\vec{B}\hat{Q}$, $A^* = \hat{P}A\hat{P}^{-1}$, and $B^* = \hat{P}B\hat{P}^{-1}$ (see Zeng, 2008).

2.2. Meanings of final output rate and input multiplier

It is known that the intermediate output rate plus the final output rate equals 1, namely, $\vec{A}E + Y = E$. The intermediate output is to be expended for producing the final output. As compared with the intermediate output, the final output is the only product that can be of material benefit to the people. Also, the final purpose of production is exactly to satisfy the people's final material demand, i.e. to gain the final output. Thus the higher the final output rate is (viz. the lower the intermediate output rate is), the more beneficial the achievement of the ultimate goal of production is. From this viewpoint, therefore, a sectoral final output rate represents this sectoral economic efficiency about final output, and the column vector of