Mathematical Social Sciences 67 (2014) 44–49

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/econbase

Stochastic stability in the Scarf economy

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HIGHLIGHTS

- Analyzes the best-known example of tâtonnement instability: the Scarf Economy.
- Introduces a novel approach using bargaining games and stochastic stability.
- Shows stochastic stability of equilibrium in the Scarf Economy.
- Bridges simulation and analytical models.
- Paves the way for a general theory of evolutionary stability of equilibrium.

ARTICLE INFO

Article history:
Received 11 November 2012
Received in revised form
10 July 2013
Accepted 13 September 2013
Available online 27 September 2013

ABSTRACT

We present a mathematical model for the analysis of the bargaining games based on private prices used by Gintis to simulate the dynamics of prices in exchange economies in Gintis (2007). We then characterize, in the Scarf economy, a class of dynamics for which the Walrasian equilibrium is the only stochastically stable state. Hence, we provide dynamic foundations for general equilibrium for one of the best-known example of instability of the tâtonement process.

1. Introduction

The Scarf economy Scarf (1960) is the paradigmatic example of the failure of the Walrasian tâtonnement process to provide a generically valid model of the convergence of an economy to its general equilibrium. Accordingly, it has served as the test-case against which to assess alternative models of price adjustment (see e.g. Kumar and Shubik, 2004; Crockett, 2013; Ghosal and Porter, 2013, in the recent literature). More broadly, the literature has tried to address the issues raised by Scarf's example through two main lines of research. The first, in line with Scarf's own concern has focused on the issue of calculability of equilibrium (see e.g. Scarf, 1969; Herings, 1997) and more recently, somehow as a by-product of the surge of results in algorithmic game theory, on its approachability (see Papadimitriou and Yannakakis, 2010). The second has taken a more behavioral route and has seek to develop micro-foundations for out-of-equilibrium dynamics. In this respect Fisher (1983) gives an account of the initial results obtained via non-tâtonnement mechanisms, Giraud (2003) surveys approaches based on market-games, Serrano and Volij (2008) or Vega-Redondo (1997) provide evolutionary approaches, and Crockett (2013) gives a survey of experimental results.

It seems that the two strands of literature have never converged again. That is the non-tâtonnement literature has abandoned the objective of providing dynamic micro-foundations for the Walrasian equilibrium of an economy in favor of a much more modest one, which is to show convergence to a stable and generally Pareto optimal outcome. This program remains at odds with the central role assumed by the concept of general equilibrium in most of the theoretical and applied literature.

As a potential contribution to the resolution of this dilemma, the aim of this note is to exhibit, in the framework of the Scarf economy, an approach of equilibrium dynamics that is both micro-founded and well-behaved asymptotically. Given its remarkable status, we assume that the Scarf economy is a good starting point to motivate further generalizations. Our model is based on a series of recent contributions (see Gintis, 2007, 2012), which have revisited the issue of "walrasian dynamics" using computer simulations where agents repeatedly perform the following sequence of operations: they receive their initial endowment, engage in bilateral trades on the basis of private prices, consume, and update their private prices on the basis of the utility these prices yielded during the period. Put differently, we focus on evolutionary dynamics in bargaining games played by agents who use private prices as strategies. This approach harnesses the power of evolutionary dynamics both as a model of economic behavior and as a computational paradigm. As a matter of fact, Gintis (2007,

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http://dx.doi.org/10.1016/j.mathsocsci.2013.09.002

2012) report surprising results of convergence to equilibrium. We provide an analytical counterpart to these results.

Namely, we use the notion of stochastic stability (see Peyton-Young, 1993) to characterize the asymptotic properties of a stylized version of Gintis's model. We show that the general equilibrium is the only stochastically stable state of this model. This implies that independently of the initial conditions, all the agents should eventually adopt the equilibrium price and obtain equilibrium allocations.

Our result mainly builds on the assumption that out-of-equilibrium trading is efficient in the same sense as in the Hahn process (see Hahn and Negishi, 1962): after trade there are not both unsatisfied suppliers and unsatisfied demanders for any given good. We also assume that agents strategically restrict their out-of-equilibrium trade whenever it is profitable to do so. In this setting, it turns out that price movement towards equilibrium are always favorable to a majority of agents. As "price-setting power" is uniformly distributed, given that each agent has its own private price to update, this progressively leads to the general adoption of the equilibrium price.

Hence, the main contribution of the paper is to explain the behavior observed in Gintis's simulations. The formalism we develop might also pave the way for the proof of more general results of convergence to general equilibrium in evolutionary models, for which related contributions (see Serrano and Volij, 2008; Vega-Redondo, 1997) provide evidence. An important caveat however is that although our result focuses on dynamic aspects of price formation, they are valid only in a setting with steady and non-durable resources, that is in the absence of opportunities to transfer wealth across periods or to make intertemporal choices. Out-of-equilibrium dynamics in growth models certainly is an under-explored field of study.

The paper is organized as follows: in Section 2, we explicit the Markov chain structure of Gintis evolutionary bargaining models and show they are models of evolution with noise in the sense of Ellison (2000). In Section 3, we characterize out-of-equilibrium trading in the Scarf economy and give sufficient conditions on the price updating mechanism to ensure the stochastic stability of equilibrium. Section 4 offers our conclusion.

2. Evolutionary dynamics in exchange economies

We aim at investigating evolutionary dynamics in exchange economies where each agent carries a private vector of prices (i.e has a private valuation of goods), uses these private prices in order to determine acceptable trades, update them by imitating those of peers who were more successful in the trading process, and randomly mutate them in some instances.

More precisely, let us consider an exchange economy with L goods,[1] N types of agents[2] and M agents of each type.[3] All the agents have $Q := \mathbb{R}^L_+$ as consumption set. Agents of type i are characterized by a utility function $u_i : Q \to \mathbb{R}$ and a vector of initial endowment $\omega_i \in Q$. Moreover, agent (i, j) (the jth agent of type i) is endowed with a normalized vector $p_{i,j}$ of private prices chosen in a finite subset P of the unit simplex of $\mathbb{R}^L_+$, $S := \{p \in \mathbb{R}^L_+ \mid \sum_{\ell=1}^{L} p_\ell = 1\}$. The population of agents is then characterized by a vector $\pi \in \Pi = P^{M\times N}$.

Repeated bilateral trades between agents define a trading process, which allocates as a function of agents private prices the total resources of the economy. This process might involve some randomness in order to cope with rationing in out-of-equilibrium situations. In all generality, we can represent the trading process by a transition measure $\mathcal{T}$ from Π to Ξ which associates to a population of prices $\pi \in \Pi$, a probability distribution $\mathcal{T}_\pi$ on the set of allocations[4] Ξ defined as

$$\Xi = \left\{ \xi \in Q^{N\times M} \mid \sum_{i=1}^{N}\sum_{j=1}^{M} \xi_{i,j} = M \sum_{i=1}^{N} \omega_i \right\}, \tag{1}$$

where $\xi_{i,j}$ represents the allocation to the jth agent of type i.

Private prices are then updated through an imitation process: agents imitate peers of the same type taking into consideration the utility gained through trading. In all generality, we can represent this imitation process as associating to a population of prices $\pi \in \Pi$ and to an allocation $\xi \in \Xi$, a probability distribution $\mathcal{I}_{(\pi,\xi)}$ on Π (which gives the distribution of prices after updating).

We are then concerned with the dynamics of private prices generated by the sequential iteration of trading and imitation processes. That is the proocess in which: initial endowments are reinitialized at the beginning of each step, agents trade according to their private prices and update these as a function of the utility gained. This corresponds to the Markovian dynamics on Π defined by the transition matrix $\mathcal{F}$ such that

$$\mathcal{F}_{\pi,\pi'} = \int_{\xi\in\Xi} \mathcal{I}_{(\pi,\xi)}(\pi')\, \mathrm{d}\mathcal{T}_\pi(\xi). \tag{2}$$

If agents then randomly and independently mutate (i.e randomly choose a new price in P) with probability $\epsilon > 0$, the dynamics are modified according to

$$\mathcal{F}^\epsilon_{\pi,\pi'} = \int_{\rho\in\Pi} R^\epsilon_{\rho,\pi'} \mathrm{d}\mathcal{F}_{\pi,\rho} = \sum_{\rho\in\Pi} R^\epsilon_{\rho,\pi'} \mathcal{F}_{\pi,\rho} \tag{3}$$

where $R^\epsilon(\rho, \pi') = (1-\epsilon)^{MN-\delta(\rho,\pi')} \times \left(\frac{\epsilon}{|P|-1}\right)^{\delta(\rho,\pi')}$ and $\delta(\rho, \pi')$ denotes the number of mutations, that is the cardinal of the set $\{(i, j) \mid \rho_{i,j} \neq \pi'_{i,j}\}$.

The family $(\mathcal{F}^\epsilon)^{\epsilon\geq 0}$ then is a model of evolution in the sense of Ellison (2000), that is satisfies the following conditions:

1. $\mathcal{F}^\epsilon$ is ergodic for each $\epsilon > 0$,
2. $\mathcal{F}^\epsilon$ is continuous in ϵ and $\mathcal{F}^0 = \mathcal{F}$,
3. there exists[5] a function $c : P^{n\times m} \times P^{n\times m} \to \mathbb{N}$ such that for all $\pi, \pi' \in P^{n\times m}$, $\lim_{\epsilon\to 0} \frac{\mathcal{F}^\epsilon_{(\pi,\pi')}}{\epsilon^{c(\pi,\pi')}}$ exists and is strictly positive.

Condition (1) implies in particular that for each $\epsilon > 0$, $\mathcal{F}^\epsilon$ has a unique invariant distribution ψ^ϵ. A population $\pi \in P^{n\times m}$ is then called stochastically stable if $\lim_{\epsilon\to 0} \psi^\epsilon(\pi) > 0$.

This notion of stochastic stability can be used for the analysis of the stability of the equilibria of the underlying exchange economy thanks to the identification of an equilibrium price $\overline{p}$ with the population $\overline{\pi}$ such that every agent uses price $\overline{p}$ (that is such that for all (i, j), one has $\overline{\pi}_{i,j} = \overline{p}$). The equilibrium associated with the price $\overline{p}$ can then be called stochastically stable if $\overline{\pi}$ is. The interesting case is this where $\overline{\pi}$ is the only stochastically stable population which implies that $\lim_{\epsilon\to 0} \psi^\epsilon(\pi) = 1$ and that for vanishingly small perturbations the process eventually settles in $\overline{\pi}$ independently of the initial conditions, in other words converges to equilibrium.

1 Indexed by $\ell = 1 \cdots L$.

2 Indexed by $i = 1 \cdots N$.

3 Indexed by $j = 1 \cdots M$.

4 We shall assume that Ξ is endowed with the Borel σ-algebra.

5 This last point follows form the fact that the coefficients of $\mathcal{F}^\epsilon_{(\pi,\pi')}$ are polynomia in ϵ.