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Electrical conductivity and dielectric permittivity of sphere packings: Measurements and modelling of cubic lattices, randomly packed monosize spheres and multi-size mixtures

David A. Robinson[a], Shmulik P. Friedman[b,*]

[a]*Department Plants, Soils and Biometeorology, Utah State University, Logan, UT 84322-4820, Israel*
[b]*Institute of Soil, Water and Environmental Sciences, Agricultural Research Organization, Bet Dagan 50250, Israel*

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Abstract

New and literature measurements are compared for the electrical transport properties of densely packed spherical particles. Measurements of electrical conductivity in cubic lattices are presented and were found to agree closely with numerical solutions for the conductivity presented in the literature. Electrical conductivity and dielectric permittivity of dense random packings are presented. Deviation from the Maxwell/Maxwell-Garnett models is clearly observed. The deviation is considered to be due to the interacting electrical fields of neighbouring particles in a densely packed system. Both electrical conductivity and dielectric permittivity were described using a model containing a heuristic parameter that can be adjusted to account for this interaction (Sihvola and Kong, IEEE Trans. Geosci. Remote Sens. 26 (1988) 420). The heuristic parameter can range between 0 and 1, and a value of about 0.2 was found to describe both the electrical conductivity and permittivity data. A more physically rigorous model developed by Torquato (J. Appl. Phys. 58(10) (1985) 3790) also described the data for cubic lattices and random packings exceptionally well. The model was

*Corresponding author. Tel.: +972 3 968 3424; fax: +972 3 960 4017.
E-mail addresses: darearthscience@yahoo.com (D.A. Robinson), vwsfried@agri.gov.il (S.P. Friedman).

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rigorously derived containing a 3-point correlation function ζ_2 to describe the interaction due to the micro-geometry.

Keywords: Spheres; Granular media; Dielectric permittivity; Electrical conductivity; Cubic lattices; Maxwell-Garnett model

1. Introduction

The description of transport through porous media provides an intriguing physical problem. The transport problem of interest might be for example that of an electric current or the diffusion of a solute in water saturated porous media. Due to the broad applicability of solutions, transport problems have attracted some of the great names of physics (J.C. Maxwell [3] and Lord Rayleigh [4]). Batchelor [5] points out that the solution to the Laplace equation is applicable to eight similar transport problems in physics. Reviews on both the historical and contemporary aspects of this problem can be found in Refs. [6–9]. Problems of particular interest in earth science include, electrical conductivity, dielectric permittivity, thermal conductivity and diffusion of solute or gas molecules in porous media. The experimental data presented in the literature for a number of analogous transport problems are presented in Fig. 1. Measurements of the effective electrical properties (Fig. 1a and b) tend to be more consistent as they are more easily measured experimentally than diffusion coefficients (Fig. 1c and d). This paper is concerned mostly with the electrical properties of a medium, electrical conductivity or dielectric permittivity but discusses the findings in light of transport phenomena in general.

Both in material science and the earth sciences there is a need to characterise composite media. Often an effective property is measured, from which, it is hoped to predict the physical characteristics of the individual components or the composition of the material. In the earth sciences, Archie's law [10] has become well known for estimating the porosity of rocks from measurements of the effective electrical conductivity. Estimating the porosity of geologic formations is crucial in oil exploration. In vadose zone hydrology, Topp's [11] empirical formula is widely used to estimate the water content of soils and sediments from measurements of the effective dielectric permittivity. These relationships have proved to be of great utility, however, the relationships presented are not universal and depend on the granular structure of the material amongst other things.

As the limitations of the empirical formulas are reached, so there is a need for more robust physically based descriptions of composite media. Much groundbreaking work has come form Princeton [8] and the Schlumberger–Doll research labs [12,13] demonstrating the contribution of material geometry and structure to its effective transport properties. With the increased use of electromagnetic methods such as time domain reflectometry [11], capacitance probes [15] and ground penetrating radar [16], there is an increased need for more robust models, which can be applied relatively easily to interpret measurements of effective properties. The