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Mechanism for linear preferential attachment in growing networks

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Abstract

The network properties of a graph ensemble subject to the constraints imposed by the expected degree sequence are studied. It is found that the linear preferential attachment is a fundamental rule, as it keeps the maximal entropy in sparse growing networks. This provides theoretical evidence in support of the linear preferential attachment widely exists in real networks and adopted as a crucial assumption in growing network models. Besides, in the sparse limit, we develop a method to calculate the degree correlation and clustering coefficient in our ensemble model, which is suitable for all kinds of sparse networks including the BA model, proposed by Barabási and Albert.

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Keywords: Preferential attachment; Scale-free networks; Statistical mechanics of graph ensemble

1. Introduction

For last few years, interest and works have been concentrated on the small-world phenomenon and scale free behavior observed in real networks [1–5]. The WS model [6] and BA model [7] provide excellent explanations for these features, respectively.

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Especially, the BA model embodies the evolutional and dynamical process and generates a very rich phenomenology that captures many of the complex features emerging in the analysis of real networks. Much attention has been focused on proposing models to reproduce the observed scale-free distribution based on the variation of BA model [8–10].

In BA model [7], two ingredients leading to the scale-free distribution are growth and linear preferential attachment. As a matter of fact, the preferential attachment, which indicates that the likelihood of receiving new edges increases with the node's degree, is a central ingredient of all models to generate scale-free networks. Some authors measured the preferential attachment effect in real networks. e.g. Jeong, Néda, Barabási et al. [11], have measured the preferential attachment effect in four real networks—the citation network of articles, the Internet, the co-authorship network of researchers, and the actor collaboration network. It is found that the first two networks have linear preferential attachment, and the latter two networks have sublinear preferential attachment. It is reported that numerous models or real networks incorporate the linear preferential attachment [12,13].

But where does the preferential attachment come from? Recently, several papers have offered promising proposals and models that shed some light on this issue [14,15], but a universal answer is still lacking. In this paper, we try to give answer to this question through studying the structure of complex networks from another point of view—statistical mechanics, which is a well-founded general theory with true predictive power. We put the case of sparse limit as keystone in our work, and reveal the structure of growing networks with linear preferential attachment by the special connection probability. The BA model [7], as a highly influential model in the complex network field and typically has a linear preferential attachment, deserves our notice for its topological structure when we inspect it from the novel view.

In particular, we develop a method using the connection function to calculate the degree correlation and clustering coefficient in our ensemble model. These analytical predictions are tested by numerical simulation in BA model. Other models or real networks incorporating the sparse and linear preferential attachment characteristics can also be studied by using this method.

2. Partition function and connection probability

We investigate the structure of complex networks, basing on the exponential random model [16,17], which are a class of graph ensembles of fixed vertex number defined by analogy with the Boltzmann ensemble of statistical mechanics and can be derived from the first principle of maximum entropy. Considering that there is one link at most between each vertex pair, we map the topological structure of complex network into the equilibrium Fermi gas, with each vertex pair corresponding to a special state of Fermi gas and the number of edges between vertex pair representing the number of particle on the state. As the degree sequence $\{k_i\}$ is a set of microcosmic quantities, it can be used to explore the microcosmic structure of a network. According to the exponential random graph model [18,19], network with