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# Analytic study of clustering in shaken granular material using zero-range processes

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**Abstract**

We show that models used to describe granular clustering due to vertical shaking belong to the class of zero-range processes. This correspondence allows us to derive analytically in the large particle number limit in a very easy and straightforward manner a number of properties of the models like particle distribution functions, phase diagram, and characteristic time of clusterization.


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## 1. Introduction

Non-equilibrium phase transitions were observed in many simple systems [1–3]. Recently, it was also found that shaken granular material exhibits clustering depending on the shaking strength [4]. In his paper, Jens Eggers suggested a model for the description of the clustering of vertically shaken granular material.

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Originally, it was introduced for the two-box setup. Later the model and its modified version were used to describe experiments having more boxes [5–11].

The analytic studies of the above-mentioned papers were difficult and in many cases gave results only for specific values of compartment or particle number. In the case of exclusion models [12] and scale-free networks [13], it was proved that the correspondence to an already solved model, namely the zero-range process (ZRP) [14,15] can be of great help, as many results can be obtained directly. The aim of this paper is to analytically derive for the general case the stationary probability distribution, phase diagram using the above correspondence. We also study the dynamical properties of the system, namely the condensation process, and calculate the coarsening time.

## 2. The model

The experimental setup is as follows. The system consists of a container separated into boxes by walls which are open upwards. The whole system is then vertically shaken and particles can hop above the walls from one compartment to the other. The model of Eggers [4] defines particle fluxes between the boxes based upon the physical properties of the system like shaking strength and local particle density, etc. Two different steady states were found, both in the experiments and in the model: a homogeneous state where the boxes held roughly equal number of particles, and a condensed steady state where one compartment contains nearly all particles.

We first reiterate the definition of the homogeneous zero-range process following [14]: we consider a one-dimensional finite lattice of $L$ sites and periodic boundary conditions. The total particle number is denoted by $N$.

The dynamics of the system is given by rates $u(n)$, at which a particle leaves a site. The hopping rates $u(n)$ depend *only* on the number of particles on the site of departure and external parameters but independent of the properties of the target site. Here, we consider only the symmetric case when the hopping to the left or to the right is equally probable.

The important attribute of the ZRP is that it yields a steady state described by a product measure. That is, the steady state probability $P(\{n_\mu\})$ of finding the system in configuration $\{n_1, n_2, \ldots, n_L\}$ is given by a product of factors $f(n_\mu)$ that are called marginals

$$P(\{n_\mu\}) = \frac{1}{Z(L,N)} \prod_{\mu=1}^{L} f(n_\mu) , \tag{1}$$

where $Z(L,N)$ is the normalization factor. For the zero-range process, $f(n_\mu)$ is given by

$$f(n) = \begin{cases} \prod_{m=1}^{n} \dfrac{1}{u(m)} & \text{for } n \geqslant 1 , \\ 1 & \text{for } n = 0 . \end{cases} \tag{2}$$

The marginals are defined up to a multiplicative factor.