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On the formation of degree and cluster-degree correlations in scale-free networks

Xin Yao[a,*], Chang-shui Zhang[a], Jin-wen Chen[b], Yan-da Li[a]

[a]State Key Laboratory of Intelligence Technology and System, Department of Automation, Tsinghua University, Beijing 100084, PR China
[b]Department of Mathematics, Tsinghua University, Beijing, 100084, PR China

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Abstract

The *cluster-degree* of a vertex is the number of connections among the neighbors of this vertex. In this paper we study the cluster-degree of the *generalized Barabási– Albert model* (GBA model) whose exponent of degree distribution ranges from 2 to ∞. We present the mean-field rate equation for clustering and obtain analytically the degree-dependence of the cluster-degree. We study the distribution of the cluster-degree, which is size dependent but the tail is kept invariant for different degree exponents in the GBA model. In addition, for the degree dependence of the clustering coefficient, very different behaviors arise for different cases of the GBA model. The physical sense of the invariance property of cluster-degree is explained and more general cases are discussed. All the above theoretical results are verified by simulation.

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*Corresponding author.
E-mail address: yaoxin99@mails.tsinghua.edu.cn (X. Yao).

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1. Introduction

In recent years there has been extensive study on complex networks. The *scale-free networks*, defined as the networks whose degree distributions have the power-law form, have been the focus of a great deal of attention in the literature [1–3]. In the real world, there are a variety of scale-free networks, including citation networks [4,5], WWW [6,7], the Internet [8–10] and metabolic networks [11,12]. Barabási and Albert presented their celebrated model (*BA model*) [1,7,13] with the mechanism called *linear preferential attachment*. This model gave an explanation for the power-law degree distribution. Therefore, the BA model has been preferred as a model for the real-world networks [2,14–17]. The analytical results [18] show that the distribution exponent of this model is $\gamma = 3$. The *generalized Barabási–Albert model* [1,14,17,19], called the GBA model in this paper, introduces the offset k_0. The evolving mechanism of the GBA model is described as follows. At each time step, a vertex with m edges is added to the network. The probability of attachment of a new edge to a vertex of degree k is proportional to $k + k_0$, where the offset k_0 is a constant which can fall anywhere in the range $-m < k_0 < \infty$. This generalized model yields the power-law-form degree distribution with exponent $\gamma = 3 + (k_0/m)$, see VII.C of Ref. [17] for details. The distributions of degree and cluster-degree of GBA are illustrated in Fig. 1.

The *clustering coefficient* is another important topic in the study of scale-free networks. Let k_i denote the degree of vertex i and ℓ_i the number of connections among these k_i neighbors, which is called *cluster-degree* in this paper. The clustering coefficient of the vertex i, C_i, and the overall clustering coefficient, C, are defined below, respectively [3],

$$C_i = \frac{2\ell_i}{k_i(k_i - 1)}, \quad C = \frac{1}{N}\sum_{i=1}^{N} C_i \,, \tag{1}$$

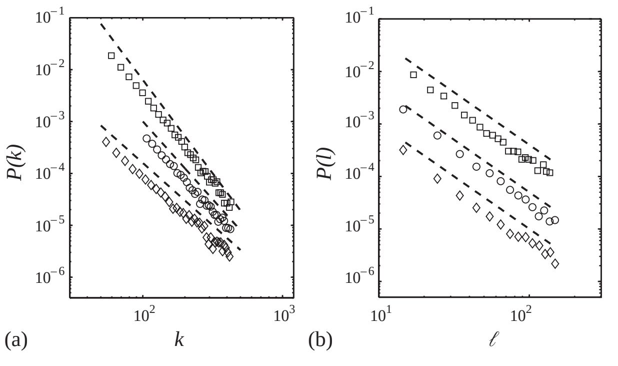

Fig. 1. (a) The degree distribution of the generalized Barabási–Albert model. (b) The cluster-degree distribution of the generalized Barabási–Albert model. In both (a) and (b), the circles are the simulating results of $m = 15$ and $k_0 = 0$, the squares are those of $m = 15$ and $k_0 = 10$, the diamonds are those of $m = 15$ and $k_0 = -10$. The dashed lines in (a) are functions of the form $f(x) \sim x^{-\gamma}$ with exponents γ being 3.6, 3 and 2.33. The dashed lines in (b) are functions $f(x) \sim x^{-\alpha}$ where $\alpha = 2$. The vertices number $N = 400\,000$ in all the simulations.