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Estimating the distribution of volatility of realized stock returns and exchange rate changes

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Abstract

Realized stock return volatility is modelled with a distribution based on the Laplace distribution. The moment properties of suggested volatility distribution, $\eta(\sigma|\lambda)$, are derived. The properties of distribution correspond to the empirical regularities found in the finance literature. ML-estimator for λ is also provided. The advantage of Laplace approach lies in estimating λ from returns distribution $f(x|\lambda)$ directly instead of volatility distribution based on bias sensitive standard deviation estimates. The goodness-to-fit tests with 5 day standard deviations of daily HEX closing price returns in period 3.1.1983–4.3.2003, daily S&P500 closing stock index returns in period 1.3.1950–27.3.2003 and daily USD/Euro exchange rate changes in period 28.12.1978–28.2.2003 support the suggested volatility distribution model.

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1. Introduction

The basic modelling approaches in finance, like the CAP-model and the Black–Scholes framework for option pricing, are based on the assumption of

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constant volatility. In practice, volatility is time dependent and many different methods are proposed to analyse the temporal evolution of volatility of asset prices. In economics the most popular has been the class of ARCH and stochastic volatility models [1,2]. In physics, the focus has been in the scaling or power-law analysis [3–5] and in distribution modelling of volatility based on lognormal and Hull and White [6] model [7,8]. The analysis of high-frequency data has opened new possibilities to less biased approximation of ex-post continuous time volatility modelling [9–12]. The main findings of this relatively new literature are that unconditional distributions of realized standard deviations and covariance of returns are highly right-skewed but the logarithmic standard deviations and correlations are approximately Gaussian. Some interesting alternatives based on Levy process and normal inverse Gaussian distributions have also been suggested [13,14]. However some more practical oriented model based approaches are still needed.

Linden [15] showed, both in theory and empirics, that Laplace (or double exponential) distribution is adequate model for stock market returns distribution. This result opens some new insights to analyse and model the distribution of volatility of stock returns and exchange rate changes [16]. Fig. 1 shows Laplace($\sqrt{2}$) and $N(0,1)$ density distributions and their right tail behaviour. Both are symmetric but excess kurtosis of Laplace distribution is evident. Thus the Laplace distribution is more peaked and has flatter tails than Normal distribution. These are the often found characteristics of (leptokurtic) financial return distributions.

The Laplace approach also opens an interesting approach to volatility modelling that is explored below in details. The structure of paper is following. Section 2 gives the derivation of volatility distribution $\eta(\sigma|\lambda)$ that depends only on one parameter (λ). ML-estimator for λ is also provided. However the main advantage of Laplace approach lies in the possibility of estimating λ from returns distribution $f(x|\lambda)$ directly instead of using volatility distribution wherein apparent bias sensitive measurements of standard deviations are needed. In Section 3 we fit in comparison to Laplace returns distribution the 5-day standard deviations of daily HEX closing price returns, daily S&P500 closing stock index returns and USD/EURO exchange rates changes to $\eta(\sigma|\lambda)$. The results are promising since ML λ estimates from $\eta(\sigma|\lambda)$ and $f(x|\lambda)$ are very close to each other. The goodness-of-fit test values for estimates of $\eta(\sigma|\lambda)$ distribution are encouraging. Section 4 closes the paper with summary of results.

2. Model

2.1. Theory

The zero location probability density function (pdf) of Laplace distribution is defined as follows:

$$f(x|\lambda) = (2\lambda)^{-1}\exp\{-|x|/\lambda\}, \quad \lambda > 0,\ -\infty < x < \infty\,, \tag{1}$$

where λ is a scale parameter and $x = \Delta \ln P_t$, i.e., the financial market return.