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The defocusing mechanism of isochronous resonances

R. Egydio de Carvalho*, G.M. Favaro

Instituto de Geociências e Ciências Exatas, Universidade Estadual Paulista, UNESP Av. 24A, 1515-Bela Vista, 13506-700 Rio Claro, SP, Brazil

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Abstract

We present a numerical study concerning the defocusing mechanism of isochronous resonance island chains in the presence of two permanent robust tori. The process is initialized and concluded through bifurcations of fixed points located on the robust tori. Our approach is based on a Hamiltonian system derived from the resonant normal form. Choosing a convenient parameter in this system, we are able to depict a comprehensive analysis of the dynamics of the problem.

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1. Introduction

Dynamical systems, in general show interesting scenarios when a perturbation parameter is allowed to vary. Two representative effects that commonly occur are bifurcations of isolated periodic orbits (or fixed points (FP)) and reconnection of isochronous resonances. The latter is typical for integrable conservative systems and

*Corresponding author.
E-mail addresses: regydio@rc.unesp.br (R.E. de Carvalho), gmfavaro@rc.unesp.br (G.M. Favaro).

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leads to a global topological rearrangement in the phase space. The former, in principle, may occur in any system controlled by a parameter. For the isochronous resonances to occur, the unperturbed system must be governed by a degenerate Hamiltonian and the rotation number must have at least an extremum or an inflection point. This is called non-twist condition, and there are many works where this scenario is present, some of these are listed in Refs. [1–8].

The bifurcations are algebraically identified through the changing of the Jacobian matrix eigenvalues as the parameter varies. In the same way, the bifurcations can be, geometrically, observed on the corresponding phase space or in the parameter space. The eigenvalues are calculated for each FP of the system and a bifurcation occurs when the sign of the real part changes (including when it is zero) or when the imaginary part disappears, or appears, as the parameter is allowed to vary. The only possible FP, for conservative systems, are the hyperbolical and the elliptical ones, whose Jacobian has null trace.

The changing of stability of a FP, continuously modifies the stability of the flux in its neighborhood, which means that it may transform the global topology of the system or produce local effects. In Ref. [8], the authors have presented a dynamical system where they analyzed some configurations of a perturbation in order to study reconnection and bifurcation. In our work, we construct a system with three island chains and two robust tori in order to explain the defocusing process of the chains. This is done by varying a parameter which allows the overlapping of the robust tori with the resonances. This article is organized as follows: in Section 2, we briefly present the resonant normal form (RNF) and the associated Hamiltonian. In Section 3, we discuss the numerical results and finally in Section 4, we present the conclusion.

2. The resonant Hamiltonian

We consider an autonomous system with two degrees of freedom around an elliptic FP through the expansion of the RNF. The formalism of the RNF is well presented in Refs. [9–11], but it essentially corresponds to an infinite power-series expansion in terms of action-angle variables, or in a similar set of variables. For general systems, this series does not converge because the resonances introduces small denominators in the expansion. To avoid this problem, we truncate the series in a desired order, such that the resulting Hamiltonian is still able to describe the means features of the original system.

The action-angle variables we use are denoted by $J_k, \theta_k, k = 1$ or 2, and our Hamiltonian is composed of two terms: first, which is called unperturbed term, depends only on the actions, and second, a remaining term (perturbation) which is θ_1-dependent. This perturbation introduces a 1:4 integrable resonance and has a second-order polynomial factor, $f(J_1)$, with two single real roots. This means that it may become null whenever it passes through a zero of $f(J_1)$, even when the perturbation parameter is not zero. As a consequence, in the phase space (J_1, θ_1), two tori appear. They are called robust tori, since they are not sensitive to the