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A falling body problem through the air in view of the fractional derivative approach

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Abstract

Recent studies have revealed that the fractional derivative can behave as a dissipative term. The analysis of a simple fractional oscillator has supported this point of view. However, other physical aspects related to fractional derivative are also important to be explored. For this purpose, in this work, we employ the fractional approach and guide ourselves by the above property to investigate the falling body problem. We show that the velocity of a falling body, in the fractional approach, can be greater ($t<1$) or less ($t>1$) than that velocity of free-fall obtained by the usual approach. Moreover, we show that the fractional derivative alone is not sufficient to attain a terminal speed. In order to provide a terminal speed into the fractional system a term, proportional to velocity, must be introduced.

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1. Introduction

Recently, the idea of employing the fractional derivative in the place of ordinary derivative has captured so much interest in the scientific community. For instance, in diffusion processes, fractional equations have been employed to describe anomalous diffusion regimes, including both subdiffusion and superdiffusion. As can be noted,

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the anomalous diffusion transport is present in diverse physical systems, e.g., it can be found in porous media [1], polymers [2], amorphous semiconductors [3] and composite heterogeneous films [4].

Despite the success of the fractional approach applied to anomalous transport, there are still several physical aspects, related to the fractional derivative, to be investigated. Recent works on the theme, have revealed that the fractional derivative can behave as a damping term [5]. In this respect, a simple fractional oscillator has also been employed to investigate the physical behavior of the fractional derivative, and it supports this point of view [6]. In order to explore some other further aspects related to the fractional derivative, one considers, in this work, a simple model involving a falling body through the air.

As it is well-known the usual approach employs the Newton second law with a dissipative term in function of the velocity of the falling body. We will show how we can incorporate the fractional derivative into the problem in order to make it clear in terms of physical aspects. To do so, our work is presented into three sections. In Section 2, we will briefly introduce the falling body problem and the usual approach to describe the process. Then, we will discuss the same problem by using the fractional approach and compare both approaches. In Section 3, we will present our conclusions.

2. The falling body problem and the fractional approach

When an object falls from rest through the air surrounding the earth, it experiences a resisting force that opposes to the relative motion in which the object moves relatively to the air. Experimentally, this resisting force is related to the relative speed v. For slow speeds the resisting force is in magnitude proportional to the speed. However, in other cases it may be proportional to the square (or some other power) of the speed. Applying the Newton second law we have

$$m \frac{\mathrm{d}v}{\mathrm{d}t} = mg - K_\alpha v^\alpha \,, \tag{1}$$

where α is considered a positive real number, K_α is a constant and g is the free-fall acceleration. For convenience, we only consider the solutions of (1) with $\alpha = 1$ and 2. In these particular cases Eq. (1) corresponds to the well-known Riccati equation [7]. For $\alpha = 1$, we obtain

$$v_1(t) = \frac{mg}{K_1} - c_1 \exp\left[-\frac{K_1}{m}\, t\right] \tag{2}$$

and

$$z_1(t) = z_{10} - \frac{c_1 m}{K_1} + \frac{m}{K_1}\left(gt + c_1 \exp\left[-\frac{K_1}{m}\, t\right]\right) , \tag{3}$$