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Physica A 348 (2005) 37–73

PHYSICA A

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On the use of the pulsed-convection approach for modelling advection-diffusion in chaotic flows—A prototypical example and direct numerical simulations

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Received 15 April 2004
Available online 30 October 2004

Abstract

This article addresses the application of pulsed system models (in which the advection operator is decoupled from the diffusion operator) for investigating the physics of dispersion/homogenization in deterministic chaotic flows. The analysis is organized along to main directions: (i) the development of a simplified time-continuous model which can be viewed as a generalization in a time-continuous frame of the baker's transformation, and which is amenable to analytical investigation, and (ii) the comparison of the results deriving from several typical pulsed-system models with the direct numerical simulation of the advection-diffusion equation. Both these approaches reveal the intrinsic ambiguity of the pulsed system approach in describing advection-diffusion problems.

PACS: 05.45.−a; 47.54.+r; 83.50.Xa; 47.52.+j

Keywords: Chaotic mixing; Pulsed systems; Advection-diffusion equation; Spectral analysis

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doi:10.1016/j.physa.2004.09.048

1. Introduction

The interaction between advection and diffusion is an important mathematical physical problem with a wealth of practical implications in fluid-dynamics [1,2], atmospheric science and pollutant dispersion [3], and chemical reaction engineering [4].

Although the quantitative formulation of this problem involves a linear parabolic equation for a scalar field $\psi(\mathbf{x}, t)$ (representing the tracer concentration):

$$\frac{\partial \psi(\mathbf{x}, t)}{\partial t} + \nabla \cdot [\mathbf{v}(\mathbf{x}, t)\psi(\mathbf{x}, t)] = \alpha \nabla^2 \psi(\mathbf{x}, t) , \quad (1)$$

the qualitative understanding of its properties is far from being fully achieved, especially in cases where the velocity field $\mathbf{v}(\mathbf{x}, t)$ gives rise to global or partial Lagrangian chaos [5]. Throughout this article we consider incompressible flows, which are defined by a solenoidal velocity field, $\nabla \cdot \mathbf{v} = 0$. This problem is particularly interesting when the diffusivity D is vanishingly small, i.e., for $\alpha \to 0$. In Eq. (1), the dimensionless number α is the reciprocal of the Peclet number Pe, $\alpha = 1/Pe = D/(LV_c)$, where D is the diffusivity, L and V_c a characteristic lengthscale and velocity associated with the flow problem, respectively.

Aside from the direct numerical simulation of Eq. (1) [6–9], a wealth of simple (one-dimensional) conceptual models for homogenization dynamics have been proposed in order to explain (at least phenomenologically) the interaction between diffusion and chaotic advection [2,10–14]. A vast class of such models is categorized under the wording of *lamellar models*, which branches itself in several archetypal formulations.

In a significant number of lamellar models, the action of advection is described exclusively in terms of a shrinking dynamics, which corresponds to the local behavior near a hyperbolic stagnation point [2,13,15]. The main limitation of this approach is essentially that it does not include the fundamental properties of advection associated with the twisting and folding of material interfaces within the mixing space. It is easy to see that modeling the folding action of an incompressible flow in a one-dimensional setting dictates that advection be represented by a non-local operator. This point has been clearly discussed by Wunsch[1] [16].

Recently, pulsed-system modeling of advection-diffusion has become a widely applied approach for the understanding of the physical properties of advection diffusion dynamics in chaotic flows [17,16,18–22]. In pulsed models, advection and diffusion act separately. These models can be viewed as a very peculiar case of Eq. (1), for which the velocity field is represented by a time-periodic repetition of Dirac's pulses.[2] This allows us to handle the resulting advection-diffusion equation by means of discrete maps (more precisely by means of the Frobenius–Perron operator

[1]To quote this Author: "..a one-dimensional incompressible velocity field would be quite dull, so we are forced to choose some other form of mixing that preserves the conservation laws that are the hallmark of incompressibility. Any model in which the advection simply rearranges the scalar field meets these requirements, but such advection is necessarily nonlocal."

[2]This formulation is very popular but can give rise to formal ambiguities. For details see Section 3.3.