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Analog study of the first passage time problem driven by power-law distributed noise

M.P. da Silva Jr., M.L. Lyra*, M.V.D. Vermelho

Departamento de Física, Universidade Federal de Alagoas, 57072-970 Maceió, AL, Brazil

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Abstract

In this work, we develop an analog circuit to generate a stochastic signal with stationary distribution exhibiting a tunable power-law tail. The proposed circuit design is a variant of a recently introduced one based on a differential equation with both multiplicative and additive noises. Here, the circuit is carefully designed in order to ensure that all components operate within their specific regimes. We provide a detailed characterization of the output signal, including the power-law exponent dependence on the tunable component. We apply the present circuit to study the first passage time problem in a simple integrate-fire model of neural dynamics driven by a non-Gaussian noise.

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Keywords: Nonlinear dynamics; Multiplicative noise; Analog simulation

1. Introduction

Non-linear dynamical systems can present a diversity of unconventional features when perturbed by an external noise [1], such as an enhancing of transport properties [2], stabilization of spatial patterns [3] and noise-induced phase transitions [4]. In

*Corresponding author. Fax: +55 82 214 1645.
E-mail address: marcelo@fis.ufal.br (M.L. Lyra).

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particular, the external noise can improve the systems response to weak external periodic pulses [5], a phenomenon termed as stochastic resonance (SR) due to its similarity with the resonance phenomena displayed by deterministic dynamical systems. SR ideas have been widely applied to better understand the behavior of many physical, chemical and biological systems, such as optical, electronic and magnetic systems [6], chemical reactions [7], as well as several features regarding neurophysiological aspects of sensory systems [8,9].

In general, studies of the above noise-induced phenomena usually considers the external noise uncorrelated in time and with random amplitudes following a Gaussian distribution function. The white noise approximation is appropriate to model systems where the time scale characterizing the relaxation of the noise auto-correlation is much shorter than the characteristic time scale of the system. However, there is a growing interest in studying dynamical systems driven by non-Gaussian noises with slowly decaying (power-law) distribution functions, once they have been observed to be quite ubiquitous in natural phenomena. One of the simplest mechanism to generate a power-law distributed noise is the random multiplicative process (RMP) [10], in which the stochastic variable is driven by a multiplicative noise. This mechanism has been widely used to model stochastic series emerging, for example, in economics [11,12] and biology [13,14]. However, a detailed comparative analysis of non-linear phenomena induced by Gaussian and power-law noises is still missing. Within this context, SR induced by colored non-Gaussian noises has been investigated in a series of recent works [15–17]. It was observed that an enhancement on the signal-to-noise ratio (SNR) can be achieved when the noise departs from Gaussian behavior. Also the peak in SNR becomes broader. The above features have been recently observed in an analog experiment using white non-Gaussian noises [18].

Analog simulation is a very useful technique in the investigation of stochastic systems. It enables, for example, large volumes of a system's parameter space to be surveyed quickly without the accumulation of truncation errors present in digital simulations. Actually, the relationship between the study of fluctuations in non-linear systems and experiments on electrical circuits is quite long-standing [19]. Although an analog simulation does not have the same accuracy of a digital one, it provides an important supplementary tool due to its relative simplicity and high speed, specially when qualitative pictures have to be established. In a recent work, Sato et al. [20] introduced an analog circuit which generates a signal with a stationary distribution with a power-law tail [21,22] based on a differential equation with both additive and multiplicative noises. Such equation models several interesting systems such as chaotic on–off intermittency [23], polymer motion in a turbulent fluid [24], non-linear coupled oscillators [25] and stock price and exchange rate changes in economics [26]. However, a critical analysis of the proposed circuit [27] has called attention to the need of a more careful circuit design in order to ensure that the operational limits of the various components are not exceeded.

In this work, we propose an alternative circuit design to generate a stationary power-law stochastic signal driven by both multiplicative and additive noises. The circuit is a variant of Sato's proposal. With the present design we provide a detailed