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Ward–Takahashi relation at finite temperature in Bose–Einstein condensation of trapped neutral atoms

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Abstract

We prove that the Ward–Takahashi relations at finite temperature are satisfied at the tree level for a system of a trapped Bose–Einstein condensate when the zero mode is included in the quasi-particle picture properly. This implies that the zero mode represents the Nambu–Goldstone mode and that it is introduced in a way consistent with the Goldstone theorem. In the proof, we employ thermo field dynamics which is a real-time operator formalism of quantum field theory at finite temperature.

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Keywords: Bose–Einstein condensation; Ward–Takahashi relation; Thermo field dynamics

1. Introduction

The Bose–Einstein condensates (BECs) of trapped neutral atoms have been created in laboratories [1–3]. Many theoretical studies on these phenomena have continued, while experimental techniques have also improved [5]. Since the atomic

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gases are dilute and interactions between atoms are weak, systems of the trapped BECs give us ideal test fields for many-body quantum physics. Especially, some clear signals of phase transition in such systems have been observed under equilibrium and non-equilibrium situations [4,6]. These and other experimental efforts will inspire theoretical studies of a challenging subject, i.e., to understand phase transitions associated with the condensate phenomena of boson.

In this paper, we investigate the Ward–Takahashi (WT) relations [7] for a trapped BEC at finite temperature. The WT relations which are among matrix elements of field operators are important because they directly reflect some very basic requirements in quantum field theory [8,9]. To derive them, we use the Noether current whose form and property are specified by the Heisenberg equation of motion and transformation properties of field operators. The Noether current is conserved when the system is invariant under the transformation considered. Next the canonical commutation relations (CCRs), combined with the Noether current, lead immediately to the WT relations. We are mostly forced to solve approximately only the Heisenberg equation which is nonlinear and complicated. It is desirable that the CCRs and symmetric properties of the system should be kept even then. This is monitored by the WT relations.

The WT relations become particularly important when the system is invariant under a continuous transformation and the symmetry is spontaneously broken. The BEC system is an example of such cases, interpreted as a spontaneous breakdown of a global phase symmetry. As is well known, when a continuous symmetry is spontaneously broken, we have the Goldstone theorem which requires the existence of the Nambu–Goldstone (NG) mode [10]. The NG mode is a gapless mode to keep the original symmetry [8]. In other words, it controls some essential dynamical properties originating from the invariance of the system. Here we emphasize the fact that the WT relations derive the Goldstone theorem [8]. If the WT relations are broken in a certain approximation, then the Goldstone theorem does not hold. Thus the WT relations are closely related to the Goldstone theorem, and this is the main reason why we check the WT relations in this paper.

There are two known formulations of quantum field theory with the operators representing lowest-energy modes in trapped BECs. They are needed to keep the CCRs. One is the zero-energy mode (ZEM) induced by the generalized Bogoliubov transformation (GBT) [11], the other is the lowest energy mode of the Bogoliubov–de Gennes (BdG) method, which is represented by the quantum coordinates (QCs) [12,13]. The relationship between both the formulations is shown in Ref. [14]. Though the quantum fields including either the ZEM or the QCs respect the CCRs, there are some differences between them. For example, while the ZEM causes infrared divergences in quantum and thermal corrections [11], one encounters no infrared divergences in the formulation of QCs [15]. It was, however, revealed that expectation values of some observables containing the QCs become time-dependent in naive calculations despite the fact that the system is in equilibrium [12,15].

Certainly the presence of either the ZEM or the QCs makes the CCRs hold in the quasi-particle representation, but it is not obvious whether the ZEM are introduced