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Of overlapping Cantor sets and earthquakes: analysis of the discrete Chakrabarti–Stinchcombe model

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Abstract

We report an exact analysis of a discrete form of the Chakrabarti–Stinchcombe model for earthquakes (Physica A 270 (1999) 27), which considers a pair of dynamically overlapping finite generations of the Cantor set as a prototype of geological faults. In this model the nth generation of the Cantor set shifts on its replica in discrete steps of the length of a line segment in that generation and periodic boundary conditions are assumed. We determine the general form of time sequences for the constant magnitude overlaps and, hence, obtain the complete time-series of overlaps by the superposition of these sequences for all overlap magnitudes. From the time-series we derive the exact frequency distribution of the overlap magnitudes. The corresponding probability distribution of the logarithm of overlap magnitudes for the nth generation is found to assume the form of the binomial distribution for n Bernoulli trials with probability $\frac{1}{3}$ for the success of each trial. For an arbitrary pair of consecutive overlaps in the time-series where the magnitude of the earlier overlap is known, we find that the magnitude of the later overlap can be determined with a definite probability; the conditional probability for each possible magnitude of the later overlap follows the binomial distribution for k Bernoulli trials with probability $\frac{1}{2}$ for the success of each trial and the number k is determined by the magnitude of the earlier overlap. Although this model does not produce the

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Gutenberg–Richter law for earthquakes, our results indicate that the fractal structure of faults admits a probabilistic prediction of earthquake magnitudes.

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Keywords: Cantor set; Overlapping fractals; Time series; Binomial distribution; Earthquake prediction

1. Introduction

Earthquakes are outcomes of fault dynamics in the lithosphere. A geological fault is comprised of two rock surfaces in contact, created by a fracture in the rock layers. The two sides of the fault are in slow relative motion which causes the surfaces to slide. However, owing to friction the surfaces tend to stick and stress develops in the regions of contact. When the accumulated stress exceeds the resistance due to friction, the fault surfaces slip. The potential energy of the strain is thereby released, causing an earthquake. The slip is eventually stopped by friction and stress development resumes. Strain continues to develop till the fault surfaces slip again. This intermittent stick–slip process is the essential feature of fault dynamics. The overall distribution of earthquakes, including main shocks, foreshocks and aftershocks, is given by the Gutenberg–Richter law [1,2]:

$$\log_{10}\mathrm{Nr}(\mathscr{M} > M) = a - b\,M\,, \tag{1}$$

where $\mathrm{Nr}(\mathscr{M} > M)$ denotes the number (or, the frequency) of earthquakes of magnitudes $\mathscr{M}$ that are greater than a certain value M. The constant a represents the total number of earthquakes of all magnitudes: $a = \log_{10}\mathrm{Nr}(\mathscr{M} > 0)$, and the value of the coefficient $b \approx 1$ is presumed to be universal. In an alternative form, the Gutenberg–Richter law is expressed as a relation for the number (or the frequency) of earthquakes in which the energy released $\mathscr{E}$ is greater than a certain value E:

$$\mathrm{Nr}(\mathscr{E} > E) \sim E^{-b/\beta}\,, \tag{2}$$

where $\beta \approx \frac{3}{2}$ is the coefficient in the energy–magnitude relation [3,4].

One class of models for simulating earthquakes is based on the collective motion of an assembly of connected elements that are driven slowly, of which the block-spring model of Burridge and Knopoff [5] is the prototype. The Burridge–Knopoff model and its variants [6,7] have the stick–slip dynamics necessary to produce earthquakes. The underlying principle in this class of models is self-organized criticality [8].

Another class of models for simulating earthquakes is based on overlapping fractals. These models are motivated by the observation that a fault surface is a fractal object [9–13]. Consequently, a fault may be viewed as a pair of overlapping fractals. Fractional Brownian profiles have been commonly used as models of fault surfaces [13–15]. In that case the dynamics of a fault is represented by one Brownian profile drifting on another and each intersection of the two profiles corresponds to