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Hydrodynamic correlation functions for a nematic liquid crystal in a stationary state

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Abstract

We show that the general procedure developed by Fox and Uhlenbeck (Phys. Fluids 3 (1970) 1893) may be employed to describe the hydrodynamic fluctuations of a thermotropic nematic liquid crystal in equilibrium and steady states. We calculate explicitly the matrix of correlation functions for the transverse variables of a nematic film confined between two horizontal plates and subjected to a constant pressure gradient. We find that the light scattering spectrum and the intensity of the Rayleigh line are calculated from the orientation correlation function in equilibrium to first order in the pressure gradient. The shape and intensity of these lines deviate from their equilibrium values by amounts proportional to the imposed gradient, leading to an asymmetry in their height and intensity. It is shown that these effects may as large as 94.3% for a value of $|\nabla p| = 2.64 \times 10^{-2}$ atm/cm of the pressure gradient, suggesting that this effect might be observable.

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1. Introduction

The Landau and Lifshitz theory of hydrodynamic fluctuations [1] close to equilibrium has been put on a firm basis within the framework of the general theory of stationary Gaussian Markov processes by Fox and Uhlenbeck long ago [2,3]. This approach has matched the theory of Onsager and Machlup [4,5] with the approach of Landau and Lifshitz, for systems where the basic state variables, $\{a_i(\vec{r},t)\}$, do not posses a definite time reversal symmetry. Although Fox and Uhlenbeck's scheme has been applied to simple fluids and their binary mixtures [6,7], its applications to other complex fluids are rather scarce.

In spite of the fact that the theory of fluctuations in nonequilibrium fluids was initiated in the late 70's [8–13] and pursued by many authors [14–19], still nowadays several questions concerning the nature of hydrodynamic fluctuations in stationary nonequilibrium states are of current active interest. One of these issues is the long-range character of these fluctuations, specially far away from instability points [20]. Thermal fluctuations in an equilibrium fluid always give rise to short-range equal time correlation functions, except close to a critical point. But when external gradients are applied, equal-time correlation functions can develop long-range contributions, whose nature is very different from those in equilibrium. For many models and systems in nonequilibrium states it has been shown theoretically that the existence of the so called generic scale invariance [21,22], is the origin of the long range nature of the correlation functions [23]. In the case of a simple fluid in a thermal gradient the structure factor, which determines the intensity of the Rayleigh scattering, diverges as q^{-4} for small values of the wave number q. This amounts to an algebraic decay of the density–density correlation function, a feature that has been verified experimentally [24–27].

However, in spite of the considerable interest in fluctuations about dissipative steady states of simple fluids during the last two decades, there are few similar studies for equilibrium or nonequilibrium stationary states of complex fluids. Among these the enhancement of concentration fluctuations in polymer solutions under external hydrodynamic and electric fields [28], or the case of a polymer solution subjected to a stationary temperature gradient in the absence of any flow [29], have been discussed. Also, the behavior of fluctuations about some stationary nonequilibrium states have been analyzed in the case of thermotropic nematic liquid crystals. Specific examples are the nonequilibrium situations generated by a static temperature gradient [30], a stationary shear flow [31] or by an externally imposed constant pressure gradient [32–34]. Although, these studies have considered only the particular case of transverse modes of the nematic, in the first two cases it was found that the nonequilibrium contributions to the corresponding light scattering spectrum were small, but in the case of a Poiseuille flow induced by an external pressure gradient the effect may be quite large. To our knowledge, however, at present there is no experimental confirmation of these effects, in spite of the fact that for nematics the scattered intensity is several orders of magnitude larger than for ordinary simple fluids.