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Chaos and dynamical transition of a single vehicle induced by traffic light and speedup

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Abstract

We study the dynamical behavior of a single vehicle moving through a sequence of traffic lights, where the vehicle speeds up to retrieve the delay induced by traffic lights and the traffic lights turn on and off periodically on a single-lane highway. The dynamical model of the vehicle controlled by both speedup and traffic light is expressed in terms of the nonlinear map. The motion of vehicle is controlled by two parameters: cycle time and speedup rate. When the speedup rate is higher than the critical value, the vehicle exhibits chaotic motion. The vehicle moves chaotically even if the model is deterministic. By varying both parameters, the complex dynamical transitions among the regular, periodic, and chaotic motions of the vehicle occur. The dynamical transitions between the complex motions are clarified.

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1. Introduction

Recently, transportation problems have attracted much attention in the field of physics [1–5]. The traffic flow, pedestrian flow, and bus-route problem have been

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studied from a point of view of statistical mechanics and nonlinear dynamics [6–23]. Interesting dynamical phase transitions have been found in the transportation systems. Jams and chaos are typical signatures of the complex behavior of traffic flow [24,25].

Mobility is, nowadays, one of the most significant ingredients of a modern society. In urban traffic, the flow is controlled by traffic lights to give priority to the road because the city traffic networks often exceed capacity. The flow throughout depends highly on both cycle time and strategy. The dynamical state of traffic changes by varying the cycle time and strategy. Brockfeld et al. have studied optimizing traffic lights for city traffic by using a CA traffic model [26]. They have clarified that the flow throughout is improved by traffic light control strategies. Sasaki and Nagatani have investigated the traffic flow controlled by traffic lights on a single-lane roadway by using the optimal velocity model [27]. They have derived the relationship between the road capacity and jamming transition.

Very recently, it has been found that there is a self-similar behavior for a single vehicle moving through an infinite series of traffic lights [28]. This is due to passing through the traffic lights without stopping at specific cycle times. Also, it has been shown that the single vehicle exhibits a chaotic behavior at a specific cycle time of traffic lights [29]. The chaotic motion is induced by the acceleration and deceleration for starting and stopping at traffic lights. The origin of chaotic motion is definitely different from that of the self-similar behavior.

Generally, a vehicle speeds up in order to retrieve the delay induced by the stopping at traffic lights because a driver wants to arrive at his destination as soon as possible. It is unknown how the speedup affects the motion of a vehicle moving through the sequence of traffic lights. Does the vehicle move periodically or chaotically? How does the tour time of the vehicle vary with the speedup and cycle time?

In this paper, we study the behavior of a single vehicle moving through an infinite series of traffic lights when the vehicle speeds up to retrieve the delay of stopping at traffic lights. We present a nonlinear-map model to describe the dynamics of vehicle traffic controlled by both speedup and traffic light. We investigate the dynamical behavior of a single vehicle by iterating the nonlinear map. We clarify the dynamical states of a single vehicle through a sequence of traffic lights by varying speedup rate and cycle time of traffic lights. We show that the dynamic state of the vehicle changes at specific values of cycle time: the vehicle behaves chaotically or periodically for short cycle times, while it moves only periodically for long cycle times.

2. Model and nonlinear map

We consider the motion of a single vehicle going through an infinite series of traffic lights. The traffic lights are numbered, from upstream to downstream, $1, 2, 3, \ldots, n$, $n+1, \ldots$. The distance between traffic lights n and $n+1$ is defined as l_n. The vehicle moves with the mean speed $v(n)$ between traffic lights n and $n+1$. We assume that the mean speed $v(n)$ increases proportionally to the stopping time at the traffic light