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# Estimating the nonextensivity of systems from experimental data: a nonlinear diffusion equation approach

T.D. Frank*, R. Friedrich

*Institute for Theoretical Physics, University of Münster, Wilhelm-Klemm-Str. 9, 48149 Münster, Germany*

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**Abstract**

We consider nonextensive systems that are related to the nonextensive entropy proposed by Tsallis and can be described by means of the nonlinear porous medium equation and the nonlinear Fokker–Planck equation proposed by Plastino and Plastino. We show how to determine the degree of nonextensivity of these systems from experimental data. Both transient and stationary cases are addressed.


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## 1. Introduction

Thermostatistics and information theory based on the Boltzmann–Gibbs–Shannon measure has found wide applicability in various disciplines ranging from solid state physics to the physics of open nonequilibrium systems [1–4]. However, as pointed out time and again in the literature, this approach is subjected to various

*Corresponding author. Tel.: +49-251-83-34922; fax: +49-251-83-36328.
*E-mail address:* tdfrank@uni-muenster.de (T.D. Frank).

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limitations (e.g., Refs. [5,6]). Recently, Tsallis suggested a generalization of thermostatistics involving the nonextensive entropy measure $S_q$ that depends on a parameter $q$, which measures the degree of nonextensivity of systems [7–9]. While for $q = 1$ we deal with extensive systems (and $S_q$ recovers the Boltzmann–Gibbs–-Shannon measure), for $q \neq 1$ we deal with nonextensive systems. The hope is that such a nonextensive extention of the Boltzmann–Gibbs–Shannon thermostatistics and information theory applies to systems for which the extensive theory fails. Indeed, nonextensive thermostatistics has been successfully applied to describe, for example, the statistics of systems with long-range interactions [10,11], chaotic systems involving fractal phase spaces [12,13], systems for which temperature fluctuations might become relevant [14–16], and systems exhibiting power-law distributions [17–19]. In this context, a crucial issue is to determine the parameter $q$. For particular systems with long-range interactions, a $q$-value of about 7 has been found [10,11]. In economics, $q$-values of about 1.5 have been reported [20,21]. For chaotic systems, $q$-values have been related to system parameters [12,13]. Furthermore, for systems subjected to temperature fluctuations $q$-values have been related to the spatial scales on which key properties of these systems are typically studied [14,22,23].

In the present manuscript, the focus is on systems that exhibit power-law distributions and can be treated within the framework of the nonextensive thermostatistics suggested by Tsallis. In particular, we assume that we deal with systems that can be described by means of the nonlinear Fokker–Planck equation proposed by Plastino and Plastino, whose stationary solutions are power-law distributions that make the entropy $S_q$ maximal under appropriate energy constraints [24]. This nonlinear Fokker–Planck equation is of particular interest because if we interpret the probability density as a particle density function $\rho(x, t)$, then the Fokker–Planck equation corresponds to the porous medium equation

$$\frac{\partial}{\partial t}\rho = Q\frac{\partial^2}{\partial x^2}\rho^q \tag{1}$$

used in hydrodynamics [25–27]. Using a data analysis technique developed for Markov diffusion processes [28–31] and the concept of strongly nonlinear Fokker–Planck equations [32], we will show in this manuscript how to determine the parameter $q$ from experimental data.

## 2. Estimating the degree of nonextensivity

### 2.1. General considerations

Let us describe a stochastic process by means of a random variable $X(t)$ that is defined on a phase space $\Omega$ and distributed like $u(x)$ at time $t = t_0$. Let $P(x, t; u) = \langle \delta(x - X(t)) \rangle$ denote the probability density of $X$. Here and in what follows, $\langle \cdot \rangle$ describes ensemble averages. We assume that the evolution of $P$ is given by a