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Velocity-dependent friction and diffusion for grains in neutral gases, dusty plasmas and active systems

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Abstract

A self-consistent and universal description of friction and diffusion for Brownian particles (grains) in different systems, such as gases with Boltzmann collisions, dusty plasmas with ion absorption by grains, and for active particles (e.g., cells in biological systems), is suggested on the basis of the appropriate Fokker–Planck equation. Restrictions for application of the Fokker–Planck equation to the problem of velocity-dependent friction and diffusion coefficients are found. A general description for these coefficients is formulated on the basis of a master equation. Relation between the diffusion coefficients in the coordinate and velocity spaces is found for active (capable to transfer momentum to the ambient medium) and passive particles in the framework of the Fokker–Planck equation.

The problem of anomalous space diffusion is formulated on the basis of the appropriate probability transition (PT) function. The method of fractional differentiation is avoided to construct the correct probability distributions for arbitrary distances, which is important for applications to different stochastic problems. A general equation for the time-dependent PT function is formulated and discussed.

The generalized friction in velocity space is determined and applied to describe the friction force itself as well as the drag force in the case of a non-zero driven ion velocity in plasmas.

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Negative friction (due to ion scattering) on grains exists and can be realized for the appropriate experimental conditions.

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1. Introduction

Interest in Brownian dynamics is conditioned by a large variety of applications: granular systems, including dusty plasmas, various objects in biological systems, physical–chemical systems, etc. For open systems and for systems with non-elastic processes the relations between the friction and diffusion coefficients and even the correct specific forms of the Fokker–Planck equation have not yet been established. Recently the friction and diffusion coefficients as functions of the grain velocity V were derived for dusty plasmas [1]. Due to ion absorption by the grains, the friction coefficient can become negative [2] for the simplest model of dusty plasmas with ion absorption without atom regeneration on the grain's surface. This implies that the problem of negative friction has to be considered for more complicated models and realistic situations in dusty plasmas, in particular taking into account the grain's mass conservation, which is essential for large times of the process, in spite of the big difference between ion and grain masses. In the present paper, we consider the specific forms of the probability transition (PT) [3] for Boltzmann-type and non-Boltzmann-type collisions to calculate and compare the velocity dependent friction and diffusion coefficients in different systems, including gases, dusty plasmas and active particles. Our main goal in this paper is to determine the velocity-dependent friction and diffusion coefficients for arbitrary grain velocity and to describe the diffusion in the coordinate and velocity spaces for different systems and conditions.

In Section 2, the Fokker–Planck equation and a self-consistent description of the velocity-dependent friction and diffusion are discussed in detail on the basis of the PT function. In Section 3, we consider in detail the PT function for Boltzmann-type collisions, in particular, for Boltzmann spheres. Friction and diffusion coefficients are calculated on the basis of the Fokker–Planck equation. The restriction for application of the Fokker–Planck equation to the fast motion of grains for Boltzmann-type collisions and the necessity to use an appropriate master equation with prescribed PT function is shown. The general formulae for the velocity-dependent friction and diffusion coefficients in the velocity space are established on the basis of the master equation. In Section 4, the absorption collisions for ions by grains are considered for dusty plasmas with a high grain charge. In the framework of the simplest model with a fixed mass of grains the phenomenon of negative friction is found. The friction coefficient changes sign for some value of the grain