Available online at www.sciencedirect.com
SCIENCE DIRECT®
Physica A 347 (2005) 673–682
PHYSICA A
www.elsevier.com/locate/physa

Self-similar behavior of a single vehicle through periodic traffic lights

Takashi Nagatani*

Division of Thermal Science, Department of Mechanical Engineering, Shizuoka University, Hamamatsu 432-8561, Japan

Received 28 June 2004
Available online 8 September 2004

Abstract

We study the dynamical behavior of a single vehicle moving through a sequence of traffic lights on a single-lane highway, where the traffic lights turn on and off periodically with the synchronized or green-wave strategies. The dynamical model of the vehicle controlled by traffic lights is expressed in terms of the nonlinear map. The tour time of the vehicle exhibits a complex behavior, interacting with the sequence of traffic lights. The dynamical behavior of signal traffic at a low density changes at $t_s = l/v + \tau$, where l is the distance between the traffic lights, v the mean speed of the vehicle, and τ the delay of traffic light at the green-wave strategy. When the cycle time t_s of traffic lights is less than $(l/v + \tau)$, the vehicle exhibits the self-similar behavior, while it shows the periodic behavior for $t_s > (l/v + \tau)$. The characteristics of the self-similar behavior is clarified.

PACS: 89.40.+k; 05.45.−a; 82.40.Bj

Keywords: Transportation; Traffic light; Self-similarity; Nonlinear map

1. Introduction

Mobility is nowadays one of the most significant ingredients of modern society. Recently, transportation problems have attracted much attention in the field of

*Fax: +81-53-478-1048.
E-mail address: tmtnaga@ipc.shizuoka.ac.jp (T. Nagatani).

0378-4371/$ - see front matter © 2004 Elsevier B.V. All rights reserved.
doi:10.1016/j.physa.2004.08.007

physics [1–5]. The traffic flow, pedestrian flow, and bus-route problem have been studied from a point of view of statistical mechanics and nonlinear dynamics [6–23]. The interesting dynamical phase transitions have been found in the transportation system. The jams and chaos are typical signatures of the complex behavior of traffic flow [24,25].

In urban traffic, the flow is controlled by traffic lights to give priority for a road because the city traffic networks often exceed the capacity. Brockfeld et al have studied optimizing traffic lights for city traffic by using a CA traffic model [26]. They have clarified that the flow throughout is improved by traffic light control strategies. Also, they have shown that the city traffic controlled by traffic lights can be reduced to a simpler problem of a single-lane highway. Sasaki and Nagatani have investigated the traffic flow controlled by traffic lights on a single-lane roadway by using the optimal velocity model [27]. They have clarified the characteristics of the traffic in the typical strategies of signal control and derived the relationship between the road capacity and jamming transition.

The flow throughout depends highly on both the cycle time and strategy. The dynamical state of traffic changes by varying the cycle time and strategy. Until now, one has studied the periodic traffic controlled by a few traffic lights. It has been concluded that the periodic traffic does not depend on the number of traffic lights [26,27]. Few works deal with vehicle traffic moving through an infinite series of traffic lights.

In this paper, we study the traffic of a single vehicle moving through an infinite series of traffic lights, which are periodically positioned with a constant distance on a single-lane roadway and controlled by the synchronized and green-wave strategies. We present a nonlinear-map model to describe the dynamics of vehicle traffic controlled by traffic lights. We investigate the dynamical behavior of a single vehicle by iterating the nonlinear map. We clarify the dynamical states of a single vehicle through a sequence of traffic lights by varying cycle time of traffic lights. We show that the behavior of vehicle changes at a specific value of cycle time; the vehicle behaves a self-similar way for a short cycle time, while it exhibits a periodic behavior for a long cycle time.

2. Dynamics and model

We consider the motion of a single vehicle going through an infinite series of traffic lights. The traffic lights are periodically positioned with distance l. The vehicle moves with the mean speed v between a traffic light and its next light. Fig. 1 shows the schematic illustration of the single vehicle moving through a sequence of traffic lights. The traffic lights are numbered, from upstream to downstream, by 1, 2, 3,…, n, $n+1$,… . In the synchronized strategy, all the traffic lights change simultaneously from red (green) to green (red) with a fixed time period $t_s/2$. The traffic lights flip periodically at regular time interval $t_s/2$. Time t_s is called the cycle time. When a vehicle arrives at a traffic light and if the traffic light is red, the vehicle stops at the position of the traffic light. Then, when the traffic light changes from red to green,