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Transfer matrices for the zero-temperature Potts antiferromagnet on cyclic and Möbius lattice strips

Shu-Chiuan Chang[a,b], Robert Shrock[c,*]

[a]Department of Applied Physics, Faculty of Science, Tokyo University of Science, Tokyo 162-8601, Japan
[b]Physics Division, National Center for Theoretical Sciences at Taipei, National Taiwan University, Taipei 10617, Taiwan
[c]C.N. Yang Institute for Theoretical Physics, State University of New York, Stony Brook, NY 11794, USA

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Abstract

We present transfer matrices for the zero-temperature partition function of the q-state Potts antiferromagnet (equivalently, the chromatic polynomial) on cyclic and Möbius strips of the square, triangular, and honeycomb lattices of width L_y and arbitrarily great length L_x. We relate these results to our earlier exact solutions for square-lattice strips with $L_y = 3, 4, 5$, triangular-lattice strips with $L_y = 2, 3, 4$, and honeycomb-lattice strips with $L_y = 2, 3$ and periodic or twisted periodic boundary conditions. We give a general expression for the chromatic polynomial of a Möbius strip of a lattice Λ and exact results for a subset of honeycomb-lattice transfer matrices, both of which are valid for arbitrary strip width L_y. New results are presented for the $L_y = 5$ strip of the triangular lattice and the $L_y = 4$ and $L_y = 5$ strips of the honeycomb lattice. Using these results and taking the infinite-length limit $L_x \to \infty$, we determine the continuous accumulation locus of the zeros of the above partition

*Corresponding author.
E-mail addresses: scchang@phys.ntu.edu.tw (S.-C. Chang), robert.shrock@sunysb.edu (R. Shrock).

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function in the complex q plane, including the maximal real point of nonanalyticity of the degeneracy per site, W as a function of q.

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1. Introduction

The q-state Potts antiferromagnet (AF) [1,2] exhibits nonzero ground state entropy, $S_0 > 0$ (without frustration) for sufficiently large q on a given lattice Λ or, more generally, on a graph G. This is equivalent to a ground state degeneracy per site $W > 1$, since $S_0 = k_B \ln W$. There is a close connection with graph theory here, since the zero-temperature partition function of the above-mentioned q-state Potts antiferromagnet on a graph $G = G(V, E)$ defined by vertex and edge sets V and E satisfies

$$Z(G, q, T = 0)_{PAF} = P(G, q) , \tag{1.1}$$

where $P(G, q)$ is the chromatic polynomial expressing the number of ways of coloring the vertices of the graph G with q colors such that no two adjacent vertices have the same color [3–6]. Thus

$$W(\{G\}, q) = \lim_{n\to\infty} P(G, q)^{1/n} , \tag{1.2}$$

where $n = |V|$ denotes the number of vertices of the graph G and the symbol $\{G\}$ formally denotes the set $\lim_{n\to\infty} G$. The minimum number of colors that is necessary to color a graph G subject to this constraint is called the chromatic number of G, $\chi(G)$.

We represent a given strip as extending longitudinally in the x direction and transversely in the y direction, with width L_y vertices. Each strip involves a longitudinal repetition of m copies of a particular subgraph. For the square-lattice strips, this is a column of squares. It is convenient to represent the strip of the triangular lattice as obtained from the corresponding strip of the square lattice with additional diagonal edges connecting, say, the upper-left to lower-right vertices in each square. In both these cases, the length is $L_x = m$ vertices. We represent the strip of the honeycomb lattice in the form of bricks oriented horizontally. In this case, since there are two vertices in 1–1 correspondence with each horizontal side of a brick, $L_x = 2m$ vertices.

The general structure of the Potts model partition function for cyclic strips of the square lattice of width L_y, as a sum of powers of eigenvalues of a formal transfer matrix multiplied by certain coefficients $c^{(d)}$, $0 \leqslant d \leqslant L_y$, was given in Refs. [7] (see also Ref. [8]). The present authors (unaware of this finding in Ref. [7]) rediscovered the result in Ref. [9] and showed that it applies also to cyclic strips of the triangular and honeycomb lattices [10]. The coefficients are polynomials of degree d in the