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Complexity vs. stability in small-world networks

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Abstract

According to the May–Wigner stability theorem, increasing the complexity of a network inevitably leads to its destabilization, such that a small perturbation will be able to disrupt the entire system. One of the principal arguments against this observation is that it is valid only for random networks, and therefore does not apply to real-world networks, which presumably are structured. Here, we examine how the introduction of small-world topological structure into networks affects their stability. Our results indicate that, in structured networks, the parameter values at which the stability–instability transition occurs with increasing complexity is identical to that predicted by the May–Wigner criteria. However, the nature of the transition, as measured by the finite-size scaling exponent, appears to change as the network topology transforms from regular to random, with the small-world regime as the cross-over region. This behavior is related to the localization of the largest eigenvalues along the real axis in the eigenvalue plain with increasing regularity in the network.

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1. Introduction

The issue of whether increasing the complexity of a network contributes to its dynamical instability has long been debated. This 'complexity vs. stability' debate is especially acute in the field of ecology [1], as it relates to the importance of diversity

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for the long-term survival of ecosystems. However, understanding the relation between the network structure and its stability (with respect to dynamical perturbations) is crucial, as it is related to the robustness of systems as ubiquitous as power grids, financial markets, and even complex societies and civilizations [2]. Pioneering studies on the stability of networks, both theoretical [3] and numerical [4], suggested that increasing the network complexity, as measured by its size (N), density of connections (C) and the strength of interactions between coupled elements (σ), almost inevitably leads to the destabilization of any arbitrary equilibrium state of the system. This result, known as the May–Wigner stability theorem, seemed to fly in the face of conventional wisdom that higher diversity makes a system more capable of surviving perturbations and has since led to much research on the connection between network complexity and stability [5].

The May–Wigner argument [3] confines itself to analyzing the local stability of an arbitrarily chosen equilibrium point of the network dynamics. Under such constraints, the explicit dynamics at the nodes can be ignored and the stability is governed by the leading eigenvalue of the linear stability matrix **J**. As a first approximation, one can consider the network elements to be coupled randomly with each other. If the connection weights between linked nodes follow a Gaussian distribution (with mean 0 and variance σ^2), then it follows that **J** is a random matrix. Therefore, existing rigorous results on the eigenvalue distribution of random matrices can be applied, which allows one to make the assertion that the network is almost certainly stable if $\sqrt{NC\sigma^2} < 1$, and almost certainly unstable otherwise.

Objections to the May–Wigner argument have often revolved around the assumption of a randomly connected network. As pointed out by many ecologists, most networks occurring in nature are not random, and seem to have structures such as trophic levels in the predator–prey relations between different species. Some early studies seemed to suggest that introducing a hierarchical organization (e.g., by partitioning the adjacency matrix of the network into blocks [6] or by having tree structures [7]) can increase the stability of a network under certain conditions. However, no general consensus on this issue has yet been achieved.

The introduction of "small-world" connection topology [8] has allowed the possibility of having different kinds of structures in a network, other than a straightforward hierarchy of levels. Small-world networks have the global properties of a random network (short average path length between the elements) while at the local level they resemble regular networks with a high degree of clustering among neighbors. In fact, several empirically obtained food web networks have been analyzed by different research groups looking for evidence of small-world structures. Initial reports of small-world ecological networks based on the analysis of four food webs [9] have been challenged by a study based on seven food webs [10], and, more recently, by a comprehensive analysis of 16 food webs covering a wide variety of habitats [11]. The latter studies did not see significantly high clustering in most of these systems, compared to a random network.

In light of this, it is inevitable to ask oneself whether the introduction of small-world connectivity confers any advantage to the network. If the occurrence of higher-than-average clustering has no functional significance, then the occurrence of