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Out of equilibrium generalized Stokes–Einstein relation: determination of the effective temperature of an aging medium

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Abstract

We analyze in detail how the anomalous drift and diffusion properties of a particle evolving in an aging medium can be interpreted in terms of an effective temperature of the medium. From an experimental point of view, independent measurements of the mean-square displacement and of the mobility of a particle immersed in an aging medium such as a colloidal glass give access to an out of equilibrium generalized Stokes–Einstein relation, from which the effective temperature of the medium can eventually be deduced. We illustrate the procedure on a simple model with power-law behaviors.

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1. Introduction

As well-known, the dynamical properties of systems which are not far from an equilibrium state can successfully be analyzed in terms of the fluctuation–dissipation theorems (FDTs) [1,2]. As recalled for instance in Ref. [3], the meaning of these theorems is as follows. The fluctuation–dissipation theorem of the "first kind" (or first FDT) expresses a necessary condition for a thermometer in contact solely with the system to register the temperature of the bath. The fluctuation–dissipation theorem of the "second kind" (or second FDT) expresses the fact that the bath itself is in equilibrium.

None of these theorems is valid out of equilibrium. This happens in particular when the bath is constituted by an aging medium with properties evolving with its age (i.e., the time t_w elapsed since the instant of its preparation). In such a case, provided that departures from equilibrium remain weak, it has been proposed to associate to the FDT violation an age and frequency-dependent effective temperature $T_{\text{eff.}}(\omega, t_w)$ [4,5]. More precisely, the effective temperature has been introduced as a quantity modifying the relation between the dissipative part of a given generalized susceptibility and the associated correlation function. A few experimental measurements of the effective temperature of real out of equilibrium physical systems have subsequently been performed [6–9].

In the present paper, we focus the interest on the recent experiment as described in Ref. [9], in which the effective temperature of an aging colloidal glass is determined by studying the mean square displacement and the frequency-dependent mobility of an immersed probe particle. If the particle would evolve in a bath at equilibrium, the diffusion exponent characterizing its mean square displacement $\Delta x^2(t)$ would only depend on the exponent characterizing the real part $\Re e\, \mu(\omega)$ of its mobility [10]. For an out of equilibrium bath as characterized by an effective temperature $T_{\text{eff.}}(\omega)$, we showed in Ref. [11] that if the latter can be modelized by an inverse power-law of ω at small ω, the diffusion exponent depends on both exponents associated with $\Re e\, \mu(\omega)$ and $T_{\text{eff.}}(\omega)$. Then, in turn, independent measurements of the exponents characterizing $\Re e\, \mu(\omega)$ and $\Delta x^2(t)$ allow for the determination of the exponent associated with the effective temperature.

Here we present a more thorough analysis of this problem. We show how the full expression of the effective temperature can be deduced from the measurements of the particle mean square displacement and mobility. This general analysis does not rely on any specific hypothesis on the behaviors of $\Delta x^2(t)$ and $\mu(\omega)$. We show that in the aging medium, the particle mean-square displacement and mobility are linked together via an out of equilibrium generalized Stokes–Einstein relation, which itself derives from a modified Kubo formula for $\mu(\omega)$. The question of the determination of $T_{\text{eff.}}(\omega)$ is not straightforward since the modified Einstein relation defining the effective temperature solely involves $\Re e\, \mu(\omega)$.

The paper is organized as follows. In Sections 2 and 3, we recall the formulation of the equilibrium FDTs. We put the emphasis on the fact that for each one of the two FDTs, two equivalent formulations can be given. One formulation concerns the dissipative part of a given generalized susceptibility, while the other one concerns the