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# The strong-coupling expansion and the ultra-local approximation in field theory

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**Abstract**

We first discuss the strong-coupling expansion in $(\lambda\varphi^4)_d$ theory and quantum electrodynamics in a $d$-dimensional Euclidean space. In a formal representation for the Schwinger functional, we treat the Gaussian part of the action as a perturbation with respect to the remaining terms. In this way, we develop a perturbative expansion around the ultra-local model, where fields defined at different points of Euclidean space are decoupled. We examine the singularities of the strong-coupling perturbative expansion, analysing the analytic structure of the zero-dimensional generating functions in the coupling constant complex planes. We also discuss the ultra-local generating functional in a non-polynomial model in field theory, defined by the following interaction Lagrangian density: $\mathscr{L}_{\mathscr{I}\mathscr{I}}(g_1, g_2; \varphi) = g_1(\cosh(g_2\,\varphi(x)) - 1)$. Finally, we use the strong-coupling perturbative expansion to compute the renormalized vacuum energy of the strongly coupled $(\lambda\varphi^4)_d$ theory, assuming that the scalar field is defined in a region bounded by two parallel hyperplanes, where we are imposing Dirichlet–Dirichlet boundary conditions.


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## 1. Introduction

The purpose of this paper is twofold. First, to study the structure of the singularities in coupling constant complex plane, of the perturbative series in different perturbatively renormalizable models in field theory. Second, to compute the renormalized vacuum energy associated with a self-interacting scalar field, describing systems in the strong-coupling regime. Our approach to the second problem consists in the combination of two different techniques: the strong-coupling perturbative expansion, and an analytic regularization procedure.

Concerning the first problem, our starting point is based in the fact that these non-perturbative results cannot be obtained using the weak-coupling perturbative expansion. The basic idea is to perform a alternative perturbative expansion. Thus, in the context of the strong-coupling perturbative expansion, we investigate the analytic structure of the ultra-local generating functionals for the $(\lambda\varphi^4)_d$ model and quantum electrodynamics in coupling constant complex planes. To analyze the second problem, we have to go beyond the ultra-local approximation. A finite result can be obtained using an analytic regularization procedure. This work is a natural continuation of the program developed by Klauder [1], Rivers [2] and others who have been studying the strong-coupling expansion and the ultra-local generating functional in different scalar infrared-free models in field theory.

In field theory, using the standard weak-coupling perturbative expansion, the renormalized perturbative series cannot be summed up in different models. For example, for $P(\varphi)_2$ the renormalized perturbative series for any connected Schwinger function that can be obtained by a Wick ordering is divergent [3]. For the $(\lambda\varphi^4)_3$ model, a similar divergent behavior was proved by de Calan and Rivasseau [4]. A long time ago, Dyson gave an suggestive argument supporting the divergence of the perturbative series in quantum electrodynamics [5]. A negative value for the coupling constant corresponds to an unstable vacuum. Actually, in the literature, there are many results showing that the series obtained in different perturbative renormalizable theories in $d = 4$ do not converge for any value of the coupling constants of the interacting theories. Well-known theories with such problems are scalar models with a $(\lambda\varphi^4)$ self-interaction and also quantum electrodynamics. If one tries to perform a partial resummation of the perturbative series in both theories, Landau poles appear [6–8].

It is well known that the behavior of the standard perturbative series in powers of the coupling constant at large order is related to the analytic structure of Schwinger functions in the coupling constant complex plane, in a neighborhood of the origin [9–11]. Consequently, an appealing problem is the study of the behavior of Schwinger functions of different models, in the neighborhood of the origin in the coupling constant complex planes [12,13]. Owing to non-perturbative effects, the structure of the singularities of the perturbative series in different perturbatively renormalizable models in field theory cannot be obtained using the weak-coupling perturbative expansion. Our approach to the problem consists in the study of an alternative perturbative expansion in Euclidean field theory. In this new expansion, the structure of the singularities of the theory in the coupling constant complex plane