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Static and dynamic properties of the XXZ chain with long-range interactions

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Abstract

The one-dimensional XXZ model ($s=\frac{1}{2}$) in a transverse field, with uniform long-range interactions among the transverse components of the spins, is studied. The model is exactly solved by introducing the Jordan–Wigner transformation and the integral Gaussian transformation. The complete critical behaviour and the critical surface for the quantum and classical transitions, in the space generated by the transverse field and the interaction parameters, are presented. The crossover lines for the various classical/quantum regimes are also determined exactly. It is shown that, besides the tricritical point associated with the classical transition, there are also two quantum critical points: a bicritical point where the classical second-order critical line meets the quantum critical line, and a first-order transition point at zero field. It is also shown that the phase diagram for the first-order classical/quantum transitions presents the same structure as for the second-order classical/quantum transitions. The critical classical and quantum exponents are determined, and the internal energy, the specific heat and the isothermal susceptibility, χ_T^{zz}, are presented for the different critical regimes. The two-spin static and dynamic correlation functions, $\langle S_j^z S_l^z \rangle$, are also presented, and the dynamic susceptibility, $\chi_q^{zz}(\omega)$, is obtained in closed form. Explicit results are presented at $T=0$, and it is shown that the isothermal susceptibility, χ_T^{zz}, is

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different from the static one, $\chi_q^{zz}(0)$. Finally, it is shown that, at $T = 0$, the internal energy close to the first-order quantum transition satisfies the scaling form recently proposed by Continentino and Ferreira.

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Keywords: One-dimensional XXZ model; Long-range interaction; Classical-quantum crossover; Dynamic properties; Quantum phase transition

1. Introduction

Quantum transitions are induced by quantum fluctuations which occur in the limit of very low temperatures ($T \to 0$), where they dominate over thermal fluctuations responsible for inducing classical transitions [1]. The Mott transition, between extended and localized electronic states, is perhaps the best known and studied example of quantum transitions (metal–insulator transition) [2].

The observation of anomalous behaviour in magnetic materials at very low temperatures has stimulated the study of the quantum transitions in these systems [3,4]. In particular, since the transitions occur at $T = 0$, the study of magnetic chains is particularly welcome in view of the possibility of obtaining exact solutions which allow a rigorous description. This has been the main motivation for considering in this work the XXZ chain, with a uniform long-range interaction, where quantum and classical transitions are allowed and an exact solution can be obtained. Although the model has already been considered by Suzuki [5], his study was restricted to the analysis of the classical second-order transition. As it will be shown, besides first-order classical transitions, the model also presents quantum transitions of first and second-order. This model is amenable to rigorous study of the classical/quantum crossover. Since we are interested in the complete description of the critical behaviour (quantum and classical transitions of first and second-order), it will be considered again.

In Section 2 we present the solution of the model and obtain the equation of state. The classical critical behaviour is obtained in Section 3, and the quantum critical one in Section 4. The dynamic correlations in the field direction are studied in Section 5 and in Section 6 we present the dynamic susceptibility. Finally, in Section 7, we obtain the critical surface for the quantum and classical transitions, and the crossover lines separating the various critical regimes.

2. Basic results and the equation of state

We consider the one-dimensional XXZ model ($s = \frac{1}{2}$, N sites) with uniform long-range interactions among the z components of the spins. The Hamiltonian is