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Stochastic arbitrage return and its implication for option pricing

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Abstract

The purpose of this work is to explore the role that random arbitrage opportunities play in pricing financial derivatives. We use a non-equilibrium model to set up a stochastic portfolio, and for the random arbitrage return, we choose a stationary ergodic random process rapidly varying in time. We exploit the fact that option price and random arbitrage returns change on different time scales which allows us to develop an asymptotic pricing theory involving the central limit theorem for random processes. We restrict ourselves to finding pricing bands for options rather than exact prices. The resulting pricing bands are shown to be independent of the detailed statistical characteristics of the arbitrage return. We find that the volatility "smile" can also be explained in terms of random arbitrage opportunities.

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1. Introduction

It is well-known that the classical Black–Scholes formula is consistent with quoted options prices if different volatilities are used for different option strikes and

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maturities [1]. To explain this phenomenon, referred to as the volatility "smile", a variety of models have been proposed in the financial literature. These includes, amongst others, stochastic volatility models [2–4], the Merton jump-diffusion model [5], non-Gaussian option pricing models [6,7]. Each of these is based on the assumption of the absence of arbitrage. However, it is well-known that arbitrage opportunities always exist in the real world (see Refs. [8,9]). Of course, arbitragers ensure that the prices of securities do not get out of line with their equilibrium values, and therefore virtual arbitrage is always short-lived. One of the purposes of this paper is to explain the volatility "smile" phenomenon in terms of random arbitrage opportunities.

The first attempt to take into account virtual arbitrage in option pricing was made by physicists in Refs. [10–12]. The authors assume that arbitrage returns exist, appearing and disappearing over a short time scale. In particular, the return from the Black–Scholes portfolio, $\Pi = V - S\partial V/\partial S$, where V is the option price written on an asset S, is not equal to the constant risk-free interest rate r. In Refs. [11,12], the authors suggest the equation $\mathrm{d}\Pi/\mathrm{d}t = (r + x(t))\Pi$, where $x(t)$ is the random arbitrage return that follows an Ornstein–Uhlenbeck process. In Refs. [13,14] this idea is reformulated in terms of option pricing with stochastic interest rate. The main problem with this approach is that the random interest rate is not a tradable security, and therefore the classical hedging cannot be applied. This difficulty leads to the appearance of an unknown parameter, the market price of risk, in the equation for derivative price [13,14]. Since this parameter is not available directly from financial data, one has to make further assumptions on it. An alternative approach for option pricing in an incomplete market is based on risk minimization procedures (see, for example, Refs. [15–19]). Interesting ideas how to include arbitrage were developed in [20] where the Black–Scholes pricing model was discussed in a quantum physics setting.

In this paper, we follow an approach suggested in Ref. [4] where option pricing with stochastic volatility is considered. Instead of finding the exact equation for option price, we focus on the pricing bands for options that account for random arbitrage opportunities. We exploit the fact that option price and random arbitrage return change on different time scales allowing us to develop an asymptotic pricing theory by using the central limit theorem for random processes [21]. The approach yields pricing bands that are independent of the detailed statistical characteristics of the random arbitrage return.

2. Model with random arbitrage return

Consider a model of (S, B, V) market that consists of the stock, S, the bond, B and the European option on the stock, V. To take into account random arbitrage opportunities, we assume that this market is affected by two sources of uncertainty. The first source is the random fluctuations of the return from the stock, described by the conventional stochastic differential equation,

$$\frac{\mathrm{d}S}{S} = \mu\,\mathrm{d}t + \sigma\,\mathrm{d}W\,, \tag{1}$$