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A characterization for qualified majority voting rules ☆

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Abstract

We give an axiomatization for Qualified Majority Voting Rules where one of the alternative is socially chosen if it obtains the support of $100 \cdot q\%$ of the non-abstaining voters for some $q \in]0, 1[$. These voting rules do not satisfy Neutrality in general. Instead, we introduce the axiom of Coalition Permanency.

Keywords: Voting rules; Qualified majority

JEL classification: D71

1. Introduction

On May 21st, 2006, Montenegro voted in favor of its independence by referendum. The European Union had previously imposed that the independence would be obtained if more than 55% of the voters voted "yes". In many democratic countries, a law is passed by referendum if 50% of the voters agree. In France, the constitution can be modified if 60% (2/3 in many European countries) of the non-abstaining deputies and senators agree with the change. Then, when the choice is between two alternatives, many actual voting rules are qualified majority voting ones. The well-known majority voting rule, when a decision is made if 1/2 of the non-abstaining voters are in favor of it, is only a special case. Many voting rules like the montenegrin independence one or the constitutional change ones require a different ratio of voters supporting a decision for it to be socially made.

Obviously, qualified majority voting rules do not satisfy Neutrality and this is the main difference with majority voting as characterized by May (1952). The study of non-neutral voting

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rules is not new. However, most studies deal with absolute qualified majority rules. Among others, $S_{P,Q}$ rules with P, $Q>n/2$ if n is the size of the electorate[1] are cited in Moulin (1988). These rules are special cases of what we could interpret as $S_{P,Q}$ rules with $P+Q>n/2$, introduced and axiomatized in Moulin (1980).[2]

The closest study from ours is Masso and Vorsatz (2006). These authors give an axiomatization of weighted approval voting, i.e., qualified majority voting rules when the voters are allowed to cast ballots of votes. The most important differences between this study and ours is that we consider the sets of voters and of alternatives as given. Hence, we do not have any consistency axiom.

The purpose of this article is to give a characterization of qualified majority voting rules in general. Like majority voting, these rules are anonymous and monotonic as defined by May (1952). However, as already said, unlike majority voting, these rules are generally not neutral. Instead, we will state the "coalition permanency" property for qualified majority voting rules: if a set of abstainers can modify the outcome in favor of an alternative, then they still can do so if all the other voters abstain. We will also state the very weak "general abstention" property: if no one votes, the society abstains. We will show that qualified majority voting rules are the only ones to be anonymous, monotone and to satisfy both "coalition permanency" and "general abstention" properties.

Notice that in the literature, the most cited case of qualified majority voting rule is the voting rule used by the EU Council of Ministers. However, this rule is named qualified because it is not generally neutral but also and more importantly, because it not anonymous. Indeed, the votes of all the countries are weighted differently. We do not deal with these voting rules in this study.

In Section 2, we give the formal framework. In Section 3, we give the axioms that characterize qualified majority voting rules. The proof of our theorem is given in Appendix.

2. Formal framework

Let $N=\{1,\ldots, n\}$ be the set of individuals in the society. Each individual expresses his choice between two alternatives, x and y. The vector of all the individuals' decisions is a voting configuration, $V=(V_1,\ldots,V_n)\in\{-1,0,1\}^n=\mathcal{V}$. If $V_i=1$ (resp. $V_i=-1$), individual i votes for x (resp. y). If $V_i=0$, individual i abstains. For the sake of simplicity, we will denote $\mathcal{V}^*=\mathcal{V}\diagdown\{(0,\ldots,0)\}$, the set of all voting configuration except the one where all the individuals abstain. Let V, $V'\in\mathcal{V}$, we write $V>V'$ if and only if $V_i\geq V'_i$ for all $i\in N$ and $V_i>V'_i$ for some $i\in N$.

Let V be a voting configuration and σ be a permutation of N. The voting configuration V_σ is defined by $V_\sigma=(V_{\sigma(1)},\ldots, V_{\sigma(n)})$. For $V\in\mathcal{V}$, we define $n^+(V)=\#\{\in N/V_i=1\}$, $n^-(V)=\#\{\in N/V_i=-1\}$, $n^0(V)=\#\{\in N/V_i=0\}$ where $\#J$ is the cardinality of the set J.

Given a voting configuration V, an individual $i\in N$ and $k\in\{-1,0,1\}$, the voting configuration $V+(i,k)$ is defined by $V+(i,k)=(V_1,\ldots, V_{i-1}, k, V_{i+1},\ldots, V_n)$. Then, $V+(i,k)$ is the voting configuration obtained from V when individual $i\in N$ changes his vote for $k\in\{-1,0,1\}$.

A decision rule C is a mapping from $\mathcal{V}$ onto $\{-1,0,1\}$. Then, $C(V)$ is the alternative chosen by the society when V is the voting configuration. The decision rules we are interested in this paper are qualified majority voting rules. Let $q\in]0,1[$. As already said in the Introduction,

[1] Alternative x is elected if and only if more than P voters support it whereas alternative y is elected if and only if more than Q voters support it with P and Q not necessarily equal.

[2] Notice that more recently, Aşan and Sanver (2006) have characterized $S_{P,Q}$ rules with an adaptation of Moulin's (1980) axioms and adding Neutrality. A more general study and in a slightly different framework is given in Barberà et al. (1991). Other examples can be found in Fishburn (1973).