Mathematical Social Sciences 76 (2015) 73–86

Contents lists available at ScienceDirect

Mathematical Social Sciences

journal homepage: www.elsevier.com/locate/econbase

Symmetric majority rules ☆

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HIGHLIGHTS

- We study rules to aggregate strict rankings into a unique strict ranking.
- We deal with rules satisfying symmetries and obeying the majority principle.
- We find necessary and sufficient conditions for the existence of those rules.
- Anonymity, neutrality and reversal symmetry are considered.
- Group theory is used as the main tool.

ARTICLE INFO

Article history:
Received 18 March 2014
Received in revised form
2 April 2015
Accepted 7 April 2015
Available online 21 April 2015

ABSTRACT

In the standard arrovian framework and under the assumption that individual preferences and social outcomes are linear orders on the set of alternatives, we suppose that individuals and alternatives have been exogenously partitioned into subcommittees and subclasses, and we study the rules that satisfy suitable symmetries and obey the majority principle. In particular, we provide necessary and sufficient conditions for the existence of reversal symmetric majority rules that are anonymous and neutral with respect to the considered partitions. We also determine a general method for constructing and counting those rules and we explicitly apply it to some simple cases.

1. Introduction

Committees are often required to provide a strict ranking of a given family of alternatives. There are many procedures that members of a committee can conceive to aggregate their preferences on alternatives into a strict ranking of these alternatives. Among them the ones satisfying the principles of anonymity and neutrality are usually preferred. The principle of anonymity is the requirement that the identities of individuals are irrelevant to determine the social outcome. The principle of neutrality is instead the requirement that alternatives are equally treated. Unfortunately, despite their appeal, these principles can both be satisfied by an aggregation procedure only in very special circumstances.

Consider a committee having $h \geq 2$ members whose purpose is to strict rank $n \geq 2$ alternatives, and assume that individual and social preferences are strict rankings on the set of alternatives. A preference profile is a list of h strict rankings each of them associated with the name of a specific individual and representing her preferences. Any function from the set of preference profiles to the set of social preferences is called a rule and represents a particular decision process which determines a social ranking of alternatives, whatever individual preferences the committee members express. In such a framework, Bubboloni and Gori (2014, Theorem 5) prove that it is possible to design anonymous and neutral rules if and only if

$$\gcd(h, n!) = 1. \quad (1)$$

Condition (1), first introduced by Moulin (1983, Theorem 1, p.25), as a necessary and sufficient condition for the existence of anonymous, neutral and efficient social choice functions, is a very strong arithmetical condition rarely satisfied in concrete situations. When it fails we can only try to design rules satisfying weaker versions of the principles of anonymity and neutrality.

A possible way to weaken anonymity is to divide individuals into subcommittees and require that, within each subcommittee,

☆ We are grateful to two anonymous referees and an anonymous associate editor for providing useful suggestions for improving the readability of the paper. In particular, one of the referees suggested an interesting link between the minimal majority principle and the method of simple majority decision (see Proposition 1 and the final remark in Section 7.1). Daniela Bubboloni was supported by GNSAGA of INdAM.

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http://dx.doi.org/10.1016/j.mathsocsci.2015.04.002
0165-4896/

individuals equally influence the final collective decision, while individuals belonging to different subcommittees may have a different decision power. Analogously, we can weaken neutrality by dividing alternatives into subclasses and assuming that within each subclass alternatives are equally treated, while allowing alternatives belonging to different subclasses to be treated differently. These versions of anonymity and neutrality are certainly natural and actually used in many practical collective decision processes. That happens, for instance, when a committee has a president working as a tie-breaker or when a committee evaluates job candidates discriminating on their gender. Indeed, in the former example committee members can be thought to be divided in two subcommittees (the president in the first, all the others in the second) with anonymous individuals within each of them; in the latter example alternatives can be thought to be divided in two subclasses (the women in the first, the men in the second) such that no alternative has an exogenous advantage with respect to the other alternatives in the same subclass.

The formalization of those new concepts is natural. In fact, given a partition of individuals into subcommittees, we say that a rule is anonymous with respect to those subcommittees if it has the same value over any pair of preference profiles such that we can get one from the other by permuting the names of individuals belonging to the same subcommittee. Given instead a partition of alternatives into subclasses, we say that a rule is neutral with respect to those subclasses if, for every pair of preference profiles such that we can get one from the other by permuting the names of alternatives belonging to the same subclass, the social preferences associated with them coincide up to the considered permutation. Of course, requiring that a rule is anonymous (neutral) is equivalent both to requiring that it is anonymous (neutral) with respect to the partition whose unique element is the whole set of individuals (alternatives), and to requiring that it is anonymous (neutral) with respect to any partition of individuals (alternatives).

Certainly, beyond anonymity and neutrality, social choice theorists identify further principles that rules should meet. The majority and the reversal symmetry principles are some of them. Roughly speaking, the majority principle requires that if a large enough amount of people prefer an alternative to another one, then the former alternative must be socially preferred to the latter one. In the literature we can find several ways to interpret that principle, such as relative majority, absolute majority, qualified majority and so on; here we focus on the minimal majority principle introduced by Bubboloni and Gori (2014). Given an integer ν, called a majority threshold, not exceeding the number of members in the committee but exceeding half of it and a preference profile, we say that a social preference is consistent with the ν-majority principle applied to the considered preference profile if the fact that an alternative is preferred to another one by at least ν individuals implies that the alternative is socially ranked over the other one. A rule is said to be a minimal majority rule if it associates with every preference profile p a social preference which is consistent with the ν-majority principle applied to p for all majority thresholds ν that do not generate Condorcet-cycles for p. The principle of reversal symmetry states instead that if everybody in the society completely changes her mind about her own ranking of alternatives, then a complete change in the social outcome occurs. It can be formally described by recalling first that, given a preference, its reversal is the preference obtained making the best alternative the worst, the second best alternative the second worst, and so on. A rule is then reversal symmetric if, for any pair of preference profiles such that one is obtained by the other reversing each individual preference, the social outcomes associated with them are one the reversal of the other.

In the present paper we analyse the rules that satisfy anonymity with respect to subcommittees and neutrality with respect to subclasses, and also obey the principles of minimal majority and reversal symmetry. At the best of our knowledge, conditions assuring the existence of those rules are not known. Some contributions related to different notions of anonymity and neutrality and their link with the majority principle are instead present in the literature. Under the assumption that there are two alternatives and assuming the possibility of indifference in individual and social preferences, Perry and Powers (2008) calculate the number of rules that satisfy anonymity and neutrality and the number of rules satisfying a restrictive version of anonymity (that is, every individual but one is anonymous) and neutrality. In the same framework, Powers (2010) further shows that an aggregation rule satisfies that restrictive version of anonymity, neutrality and Maskin monotonicity if and only if it is close to an absolute qualified majority rule. Quesada (2013) identifies instead seven axioms (among which are weak versions of anonymity and neutrality) characterizing the rules that are either the relative majority rule or the relative majority rule where a given individual, the chairman, can break the ties. In the framework of social choice functions, Campbell and Kelly (2011, 2013) show that the relative majority is implied both by a suitable weak version of anonymity, neutrality and monotonicity, as well as by what they called limited neutrality, anonymity and monotonicity. Moreover, in the general case for the number of alternatives, some observations about different levels of anonymity and neutrality can be found in the paper by Kelly (1991), who uses the language of permutations groups to discuss some open problems.

Here we follow the algebraic approach developed in Bubboloni and Gori (2014) to carry on our analysis, and we also adhere to the framework and notation used there. In that paper, which we refer to for further references on anonymity, neutrality and majority principles, the authors show how the notion of action of a group on a set can naturally and fruitfully be used to study problems concerning anonymity and neutrality. Indeed, among other things, they prove that condition (1) is necessary and sufficient for the existence of anonymous and neutral minimal majority rules.[1] In this paper we adapt that algebraic reasoning in order to treat anonymity with respect to subcommittees and neutrality with respect to subclasses, together with reversal symmetry and minimal majority. We obtain, as our main result, the following theorem.[2]

Theorem A. *Assume that individuals are partitioned into $s \geq 1$ subcommittees with number of members $b_1, \dots, b_s$, and that alternatives are partitioned into $t \geq 1$ subclasses with number of alternatives $c_1, \dots, c_t$. Then:*

(i) *there exists a minimal majority rule that is anonymous with respect to the considered subcommittees and neutral with respect to the considered subclasses if and only if*

$$\gcd\big(\gcd(b_1, \dots, b_s), \operatorname{lcm}(c_1!, \dots, c_t!)\big) = 1; \tag{2}$$

(ii) *there exists a minimal majority rule that is anonymous with respect to the considered subcommittees, neutral with respect to the considered subclasses and reversal symmetric if and only if*

$$\gcd\big(\gcd(b_1, \dots, b_s), \operatorname{lcm}(2, c_1!, \dots, c_t!)\big) = 1. \tag{3}$$

1 See Theorem 14 in Bubboloni and Gori (2014).

2 Theorem A is a rephrasing of Theorem 15.