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Simple coalitional strategy profiles in repeated games[☆]

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HIGHLIGHTS

- Simple coalitional strategy profiles to avoid group deviations are introduced.
- Only one-shot deviations need to be checked to avoid coalitional deviations.
- The concept of Quasi Strong Perfect Equilibrium (QSPE) is analyzed.
- In the Cournot supergame the symmetric monopoly outcome can be sustained by a QSPE.

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ABSTRACT

In this paper we introduce simple coalitional strategy profiles to avoid group deviations in repeated games. In the repeated Cournot supergame we prove that it is possible to sustain the symmetric monopoly outcome by means of a variety of strategies which satisfy the requirement that no coalition (other than the grand one) will deviate in any subgame.

1. Introduction

Most of the equilibrium concepts in the literature of infinitely repeated games may suffer from a serious drawback: they do not consider the possibility of a group of players forming a coalition to deviate. Subgame perfect equilibrium strategies are defined to avoid single player deviations. Deviations of two or more players are often ignored. In a well known paper Bernheim et al. (1987) introduce the concept of Perfect Coalition-Proof Nash Equilibrium. In their own words: "It is frequently possible for coalitions of players to arrange plausible, mutually beneficial deviations from Nash agreements". In the context of an almost unknown paper on repeated games, Horniaček (1996) argues that group deviations should not be ignored, and that any deviation of any coalition (other than the grand one) must be punished by the complementary coalition. The following example illustrates this point: Let G be a Cournot supergame with five players, a linear demand function given by $p = 100 - z$ (if $z < 100$ and 0 otherwise) and a linear cost function with marginal cost $c = 40$. Let σ be a simple strategy profile[1] with a two-phase punishment such that $S^0 = \{(6, 6, 6, 6, 6); (6, 6, 6, 6, 6); \ldots\}$, $S^1 = \{(0, 10, 10, 10, 10); (6, 6, 6, 6, 6); (6, 6, 6, 6, 6); \ldots\}$ and $S^i = S^{1(i/1)}$ $(i = 2, \ldots, 5)$. $S^{1(i/1)}$ is identical to S^1 except that the roles of player 1 and player i are interchanged. It is easy to check that σ is a subgame perfect equilibrium (SPE) whenever the discount factor $\delta > 0.8$. Note that σ sustains the monopoly payoff $\pi^m = 180$, which is higher than the Cournot payoff $\pi^c = 100$. Simple strategy profiles recommend ignoring deviations by more than one player. Hence, if players 1 and 2 deviate from the collusion path S^0, the rest of the players, following σ, will remain in S^0 playing the monopoly quantity $q^m = 6$. Knowing this, players 1 and 2 can deviate from collusion using strategy $\sigma'_{\{1,2\}}$ given by: $S'^0 = \{(10.5, 10.5); (10.5, 10.5); \ldots\}$ and $S'^1 = S'^2 = \{(14, 14); (14, 14); \ldots\}$. Note that the quantity that maximizes

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[1] According to Abreu (1988), a simple strategy profile for an n-player game is determined by $n + 1$ outcome paths $(S^0, S^1, \ldots, S^n)$, where $S^i = \{q^i(t)\}_{t=1}^{\infty}$, for $i = 0, 1, \ldots, n$, that induce the following strategy profile defined inductively:

(i) Play S^0 until a player deviates singly from S^0.
(ii) For any $j \in N$, play S^j if the jth player deviates singly from S^i, $i = 0, 1, \ldots, n$, where S^i is an ongoing previously specified path. Continue with S^i if no deviations occur or two or more players deviate simultaneously.

the total payoff of players 1 and 2 given that players 3, 4 and 5 continue with $q^m = 6$ is 21. $\sigma'_{\{1,2\}}$ is a trigger strategy[2] that allows players 1,2 to sustain the payoff $220.5 > 180$ assuming that the rest of the players continue with $q^m = 6$. After some simple computations, it is obtained that neither player 1 nor player 2 will deviate from $\sigma'_{\{1,2\}}$ whenever $\delta > 0.529$. Thus, in any game with more than two players this kind of trigger strategy could always be used to sustain a two-player deviation if there is no reaction from the rest of the players. Of course this is not the only two-player strategy that can be used in this example, but it always works and this means that, in the general case, two-player deviations must be punished because otherwise deviations will take place (and no subcoalition will deviate further).

In order to punish coalitional deviations we introduce simple coalitional strategy profiles which generalize the simple strategy profiles defined by Abreu (1988). A simple coalitional strategy profile consists of one cooperative path and one punishment path for each coalition other than the grand one. These strategies are defined as follows: Start the cooperative path and remain on it if no player deviates. If, a coalition deviates after any history, then start the punishment phase of that coalition. Only deviations of all players are ignored.

The equilibrium concept used throughout the paper is the Quasi Strong Perfect Equilibrium (QSPE) introduced by Horniaček (1996). A strategy profile is a QSPE if no coalition can, taking the actions of its complement as given, deviate in a way that benefits all of its members. It is explained in Section 3 why the Strong Perfect Equilibrium of Rubinstein (1980) cannot be used.

Next, we outline why the problem of checking whether a simple coalitional strategy profile is a QSPE can be so complex especially when the number of players n is big. To avoid coalitional deviations we need to punish all coalitions except the grand one. Even in the simplest case of a single punishment for each coalition (irrespective of the phase in which the deviation takes place) deviations of any of the $2^n - 2$ coalitions from any of the $2^n - 1$ outcome paths must be avoided. Each coalition could deviate for only one period, for any finite number of periods, or even forever. Furthermore, coordinated deviations (which are explained in detail in Section 3) must be taken into account. As will be shown, these coordinated deviations could potentially be infinitely complex.

A relevant contribution of this paper is to simplify this problem substantially. In Section 3 we generalize a result similar to that of Abreu in 1988. We prove that only one-shot deviations need to be checked to avoid coalitional deviations, where a one-shot deviation is a single-period deviation followed by sticking to the strategy in subsequent periods.

To obtain all the major results of this paper we need to introduce an auxiliary equilibrium concept which is even stronger than QSPE, and which we call the Quasi Even Stronger Perfect Equilibrium (QESPE). A strategy profile is a QESPE if no coalition other than the grand one, taking the actions of its complement as given, can deviate in a way that increases the sum of the payoffs of all of its members. Note that if a strategy is a QESPE then it is also a QSPE, whereas the reverse is not true.

Another contribution of this paper is to show that in the Cournot supergame with any number of players it is possible to sustain the symmetric monopoly outcome by means of a variety of strategies which satisfy the requirement that no coalition other than the grand one may deviate in any subgame (provided that the discount factor is close enough to 1). A straightforward conclusion from this result is that, at least in the symmetric Cournot model, any coalition which has the possibility of improving the payoffs of all of its members via a deviation, also has different strategies for sustaining that deviation in a credible way (where credibility means that no subcoalition will deviate further). This enables us to conclude that any deviation of any coalition must be punished by the complementary coalition.

The rest of the paper is organized as follows. Section 2 contains the preliminaries. Sections 3 and 4 present the results. Section 5 concludes with some comments on related work, with special attention to Horniaček (1996). The Appendix is divided into two parts, one with the lemmas used in the paper and their proofs and the other with the proofs of the results.

2. Preliminaries

Let $G = (Q_1, \ldots, Q_n; \pi_1, \ldots, \pi_n)$ be an n-player game where $N = \{1, \ldots, n\}$ is the set of players, Q_i is the set of actions q_i of player i and $\pi_i : Q = Q_1 \times \cdots \times Q_n \longrightarrow R$ is player i's payoff function.[3]

The associated infinitely repeated game with discounting is denoted by $G^\infty(\delta)$ where $\delta \in (0, 1)$ is the discount factor. If $q(t) = (q_1(t), \ldots, q_n(t))$ is the vector of actions played in period t, then $\{q(1), \ldots, q(t)\}$ is a history h of length t. A *strategy* σ_i of player i in $G^\infty(\delta)$ is a sequence of functions σ_i^t (or $\sigma_i(t)$) from the set of all histories of length $t-1$ to Q_i, so $\sigma_i^1 \in Q_i$ is the initial action of player i. A stream of action profiles $\{q(t)\}_{t=1}^\infty$ is referred to as an outcome path and is denoted by S. A strategy profile $\sigma = (\sigma_i)_{i\in N}$ generates an outcome path $S(\sigma) = \{q(\sigma)(t)\}_{t=1}^\infty$ defined inductively by:

$$q(\sigma)(1) = \sigma^1$$
$$q(\sigma)(t) = \sigma^t(q(\sigma)(1), \ldots, q(\sigma)(t-1)), \quad \text{if } t > 1.$$

The value $\pi_i(q(t))$ denotes the payoff of player i in period t when the outcome in this period is $q(t)$. And $\Pi_i(S)$ denotes the discounted payoff of player i for the outcome path $S = \{q(t)\}_{t=1}^\infty$:

$$\Pi_i(S) = \sum_{t=1}^{\infty} \delta^{t-1}\pi_i(q(t)).$$

$\Pi_i(S, t)$ is the sum of discounted payoffs of player i from the outcome path S starting from its t-th period.

The discounted payoff of player i in $G^\infty(\delta)$ obtained with the strategy profile σ is $\Pi_i(\sigma) = \Pi_i(S(\sigma))$. $\Pi_i(\sigma \mid h)$ denotes the discounted payoff of player i when $(\sigma \mid h)$ is the continuation of σ after h.

A *coalition* D is a nonempty subset of N. Let $Q_D = \Pi_{i\in D} Q_i$, $Q_{-D} = \Pi_{j\in N\setminus D} Q_j$, $q_D = (q_i)_{i\in D} \in Q_D$ and $q_{-D} = (q_j)_{j\in N\setminus D} \in Q_{-D}$. We denote by $\sigma_D = (\sigma_i)_{i\in D}$ a strategy of coalition D and by $\sigma_{-D} = (\sigma_j)_{j\in N\setminus D}$.

In this paper, we consider the Cournot Supergame with perfect monitoring. So the preliminaries for this model are also introduced here.

Take n firms producing a homogeneous product at a constant marginal cost $c > 0$. The industry inverse demand function is denoted by $p(z)$ and the payoffs are $\pi_i(q_1, \ldots, q_n) = (p(q_1 + \cdots + q_n) - c)\, q_i$, where q_i is the output of firm i.

Some reasonable assumptions about this game are:

Assumption A$_1$. $p : R_+ \longrightarrow R_+$ is continuous, differentiable and with $p'(z) < 0$ for all $z > 0$ such that $p(z) > 0$, $\lim_{z\to\infty} p(z) = 0$, and $p(0) > c$.

We introduce a capacity constraint in Q_i in order to make this set compact. Formally, $Q_i = [0, \bar{q}(\delta)]$ for all $i = 1, \ldots, n$, where

[2] A Grim-trigger strategy prescribes cooperating to begin with and as long as the others cooperate; if any of the others deviates it recommends switching to the one-shot equilibrium strategy and playing it forever.

[3] We assume that the set of payoffs $\{\pi(q) | q \in Q\}$ is bounded where $\pi \equiv (\pi_1, \ldots, \pi_n)$.