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Network formation under mutual consent and costly communication[☆]

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ABSTRACT

We consider two different approaches to describe the formation of social networks under mutual consent and costly communication. First, we consider a network-based approach; in particular Jackson–Wolinsky's concept of pairwise stability. Next, we discuss a non-cooperative game-theoretic approach, through a refinement of the Nash equilibria of Myerson's consent game. This refinement, denoted as monadic stability, describes myopically forward looking behavior of the players. We show through an equivalence that the class of monadically stable networks is a strict subset of the class of pairwise stable networks that can be characterized fully by modifications of the properties defining pairwise stability.

1. Costly network formation under mutual consent

The theory of network formation has been extensively studied by economists and game theorists in the past decade. Following the seminal contribution of Jackson and Wolinsky (1996) that initiated the game theoretic literature on network formation, a relatively sparse strand in this literature has addressed the modelling of mutual consent in link formation. This realistic criterion requires that both parties actively communicate their agreement to the formation of a link between them. We make a contribution to this literature by investigating myopic forward-looking behavior under costly communication in the context of a non-cooperative network formation game.

Myerson (1991) already considered a purely non-cooperative approach to network formation under mutual consent—the so-called "consent game". In this normal form game, every player sends messages to all other players with whom she wants to form a link. The links formed are exactly those for which both players indicate their willingness to establish it. Myerson pointed out that the resulting class of networks supported by Nash equilibria in the consent game is very large and, thus, there is a substantial indeterminacy problem concerning the non-cooperative approach to network formation under mutual consent.

In this paper we confirm this assessment for costly communication. Under strictly positive communication costs, the empty network is always supported through a strict Nash equilibrium in the consent game, in which no player sends a message to any other player. Thus it is quite likely that myopic, selfish behavior may not lead to the formation of meaningful, non-trivial social networks.

Next, within Myerson's consent game we develop a belief-based equilibrium concept — denoted as *monadic stability* — for understanding a purely non-cooperative process of network formation. To investigate the relationship between Jackson and Wolinsky's concept of pairwise stability and this non-cooperative process of network formation, we explicitly assume that (i) players use minimal information about the payoffs, and (ii) players are boundedly rational. Unlike other models of strategic network

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formation, players need not be aware of the payoffs of all other players associated with every other network. For any given network g, a player only needs to know the payoffs associated with the *adjacent* networks, i.e.; she only needs payoff information concerning any change (creation or deletion) regarding her direct links in g.

In order to make decisions, players form simple myopic beliefs how other players will behave based on the benefits these players expect to receive from their direct links. According to these myopic beliefs, each player i assumes that another player j is willing to form a new link with i if j stands to benefit from it in the prevailing network. Similarly i also assumes that j will break an existing link ij in the prevailing network if j does not benefit from having this link.

What really makes these beliefs myopic is the fact that in this process player i assumes that all other links in the prevailing network remain unchanged. In other words, when evaluating one link, players do not take into account the consequences of modifying that link for benefits from other links for themselves and other players. In our model, agents play a best response to their myopic beliefs about what other players will do. Interestingly, we find that the class of monadically stable networks in the consent game under costly communication is a well defined subset of pairwise stable networks. Thus, it can be argued that the introduction of simple myopic beliefs overcomes the unwillingness to form links induced by the costly nature of communication and the selfishness incorporated into the Nash equilibrium concept within Myerson's consent game.[1]

Our paper represents a first attempt to explore the relationship between non-cooperative game-theoretic models of network formation and network-based considerations. We find that relatively little information about payoffs coupled with myopic reasoning about the consequences of link formation leads to a class of highly desirable pairwise stable networks. This contrasts with results based on advanced reasoning about link formation, found in the work on farsightedness in network formation. (Dutta et al., 2005; Page et al., 2005; Herings et al., 2009).

Future work should further explore the role that is played by larger, but imperfect amounts of information in network formation. It should be clear from our findings that this research should also focus on introducing belief structures that are more sophisticated in nature. More advanced reasoning will enable us to get rid of one problematic feature of monadic stability—its possible non-existence.

2. Models of network formation under mutual consent

In this section we introduce the basic concepts and notation pertaining to networks. We follow the notation and terminology set out in Jackson and Wolinsky (1996), Dutta and Jackson (2003), and Jackson (2008).

Throughout we denote by $N = \{1, \ldots, n\}$ a fixed, finite player set. We limit our discussion to *non-directed networks* on the player set N. Formally, if two players $i, j \in N$ with $i \neq j$ are *linked*, we use the notation ij to describe the binary set $\{i, j\}$ that represents this undirected link. Thus, $g_N = \{ij \mid i, j \in N,\ i \neq j\}$ is the set of all potential links.

A *network* g on N is now introduced as an arbitrary set of links $g \subset g_N$. In particular, the set of all feasible links g_N itself is called the *complete network* and $g_0 = \varnothing$ is known as the *empty network*. The collection of all networks is denoted by $\mathbb{G}^N = \{g \mid g \subset g_N\}$.

The set of (direct) *neighbors* of a player $i \in N$ in the network g is given by $N_i(g) = \{j \in N \mid ij \in g\} \subset N$. Similarly, $L_i(g) = \{ij \in g_N \mid j \in N_i(g)\} \subset g$ is the *link set* of player i in g. We apply the convention that for every player $i \in N$, $L_i = L_i(g_N) = \{ij \mid i \neq j\}$ is the set of all potential links involving player i.

For every pair of players $i, j \in N$ with $i \neq j$ we denote by $g + ij = g \cup \{ij\}$ the network that results from adding the link ij to the network g. Similarly, $g - ij = g \setminus \{ij\}$ denotes the network obtained by removing the link ij from network g. This convention can be extended to sets of links: If $g \cap h = \varnothing$, we let $g + h = g \cup h$ and for $h \subset g$ we define $g - h = g \setminus h$.
We base our analysis on the hypothesis that for the formation of a link between two individuals, both have to consent explicitly to the formation of this link. We distinguish two fundamentally different approaches to the modelling of consent in link or network formation. First, one can consider equilibrium concepts based on the network structure directly, formulating a *network-based* approach.

Second, one can model link formation as the outcome of a non-cooperative game, formulating a *game-theoretic* approach. In this approach the players are driven by individual (game-theoretic) payoffs derived from the network payoff function and game-theoretic equilibrium concepts can be used to model the outcomes of such network forming behavior.

Relationship building — formalized through the link formation process — results in a network. Within a network, benefits for the players are generated depending on how they are connected to each other; thus, allowing for (widespread) externalities to network formation. Formally, for every player $i \in N$, the *network benefit function* $\sigma_i : \mathbb{G}^N \to \mathbb{R}$ assigns to every network $g \subset g_N$ a gross benefit $\sigma_i(g)$ that is obtained by player i when she participates in network g.

For every player $i \in N$, we introduce individualized *link formation costs* represented by $c_i = (c_{ij})_{j \neq i} \in \mathbb{R}_+^{N \setminus \{i\}}$. Here, for some links $ij \in g_N$ it might hold that $c_{ij} \neq c_{ji}$. Thus, the cost system c describes the difficulty of communicating between players and represents the costly nature of human interaction. Now, the pair $\langle \sigma, c \rangle$ represents the basic benefits and costs of link formation to the players in N.

From $\langle \sigma, c \rangle$ we derive for every player $i \in N$ a *network payoff function* $\varphi_i : \mathbb{G}^N \to \mathbb{R}$ by

$$\varphi_i(g) = \sigma_i(g) - \sum_{ij \in L_i(g)} c_{ij}. \tag{1}$$

The function φ assigns to each player the net proceeds from participating in a network.

2.1. Network-based stability concepts

Jackson and Wolinsky (1996) introduced the idea that equilibrium in a network formation process is based on whether participating pairs of players have incentives to delete existing links or add additional links to the network. This approach has been developed further by Jackson and Watts (2002), Jackson and van den Nouweland (2005), and Bloch and Jackson (2007).

The seminal concept in this network-based approach requires that no player has the incentive to delete an existing link and for a non-existing link no pair of players have common interests to form this link. This "pairwise stability" concept can be defined in three steps:

(i) A network $g \subset g_N$ is *link deletion proof* if for every player $i \in N$ and every link $ij \in L_i(g)$ it holds that $\varphi_i(g) \geqslant \varphi_i(g - ij)$.
(ii) A network $g \subset g_N$ is *link addition proof* if for every pair of players $i, j \in N$ with $ij \notin g$: $\varphi_i(g + ij) > \varphi_i(g)$ implies $\varphi_j(g + ij) < \varphi_j(g)$.
(iii) A network $g \subset g_N$ is *pairwise stable* if g is link deletion proof as well as link addition proof.

[1] We believe that these beliefs represent a form of confidence or trust in others in this sense. In other words when a player takes a plunge into the deep in the form of costly communication, she expects that the other player will help her out.