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Appraising diversity with an ordinal notion of similarity: An axiomatic approach ☆

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Abstract

This paper axiomatically characterizes a rule for comparing alternative sets of objects on the basis of the *diversity* that they offer. The framework considered assumes a finite universe of objects and an *a priori* given ordinal quaternary relation that compares alternative *pairs* of objects on the basis of their dissimilarity. The rule that we characterize is the *maxi–max* criterion. It considers that a set is more diverse than another if and only if the two most dissimilar objects in the former are weakly as dissimilar as the two most dissimilar objects in the later. Some connections with the issue of appraising freedom of choice are also provided.

Keywords: Diversity; Measurement; Axiomatic; Freedom of choice

JEL classification: D63; D69; Q20

1. Introduction

Would the killing of 50,000 flies of a specific species have the same impact on the reduction of biological diversity than that of 200 white rhinoceros? Is the diversity of opinions expressed in the written press larger in France than in the US? Is the choice of models of cars offered by a particular retailer more diverse than that of another? These are examples of questions whose answers require a precise notion of *diversity*.

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Biologists have been probably the first scientists interested in developing and implementing numerical indices that aim at measuring the biological diversity offered by alternative ecosystems. One of the most widely used of these indices is a generalization of Shannon's (1948) *entropy* measure proposed in biology by Good (1953) (see e.g. Baczkowski et al. (1997,1998) and Magurran (1998) for other refinements and discussions of this class of indices). This class of indices evaluates the diversity of any ecosystem by counting, for each species, the frequency of living individuals within the species relative to the total number of living individuals and calculates a weighted entropy over these relative frequencies. Yet, despite its wide use and computational convenience for applications, this index lacks sound justifications. Why after all should one use the specific entropy formula for appraising the impact of major changes on biodiversity?

Answering questions like this is important in these days where many countries who have ratified the UN 1992 convention on biological diversity have adopted economically costly environmental regulations in order to prevent a deterioration of biological diversity caused by human activities. It is all the most important as the generalized entropy measure suffers from the drawback of paying no attention whatsoever to either inter-species *dissimilarities*, or to the possibility for two individuals of the same species to be less similar than two individuals coming from different species. For instance, according to the generalized entropy formula, a world in which all living individuals are equally split between two species of fly is just as diverse as one in which the living individuals are split equally between chimpanzees and sea horses.

Efforts, often due to economists, have been made in the last 15 years to develop criteria for appraising diversity that are sensitive to the dissimilarity that may exist between living individuals. At the origin of these efforts are the contributions of Weitzman (1992, 1993, 1998) which assume the *a priori* given existence of a *cardinally meaningful* numerical distance between living creatures. Such a numerical distance enables one to say things such as "the biological distance between a chimpanzee and a bee is exactly twice that between a trout and a salmon". Using such a numerical distance, Weitzman (1992) proposes a sophisticated iterative lexicographic method for appraising the diversity offered by a set of living individuals. Using a somewhat different setting, Bossert, Pattanaik, and Xu (2003) provide an axiomatic characterization of Weitzman's method by taking as given a cardinal numerical measure of distance between the objects.

An alternative approach to diversity appraisal has been proposed by Nehring and Puppe (2002) (see also Nehring (1997) and Nehring and Puppe (2003)) who suggest defining the diversity of a set as the sum of the values of the attributes realized by the elements in the set. Nehring and Puppe (2002) justify their approach by providing a representation theorem. Specifically they identify the properties of a ranking of *lotteries over sets* of objects that are necessary and sufficient for being numerically represented by the expectation of the sum of the values of the attributes realized in the sets, for some family of attributes and some function valuing the attributes. Yet, as usual with representational constructs (see e.g. the well-known comments made by Sen on Harsanyi's justification for utilitarianism), they do not by themselves provide any reason for using the identified numerical representation rather than some other. Hence, operational use of Nehring and Puppe's (2002) approach requires the selection of both a class of relevant attributes for the objects (for instance being a mammal) and a (cardinally meaningful) function that weights the various attributes.[1]

[1]As recognized by Nehring and Puppe (2003) themselves, "the cardinal scale inherent in our concept of diversity is essential". A more detailed discussion of the literature on diversity, including the multi-attributes approach, is provided in Gravel (2007).