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Pricing American options: RNMs-constrained entropic least-squares approach

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ABSTRACT

Recently the entropy-based valuation of European options (Stutzer, 1996) has been extended to American option pricing. In this paper, we improve the pricing accuracy by incorporating informative risk-neutral moments (RNMs), which are recovered from a set of market-available option data, as constraints into the entropy framework. With these RNMs, an appropriate risk-neutral measure close enough to the correct one is achieved. An adjusted least-squares algorithm is then utilized to determine the optimal exercising strategy. The results based on simulations and empirical analysis demonstrate that our method can price American options rather accurately and significantly outperforms the benchmark methods.

1. Introduction

An entropy-based valuation method, canonical valuation, was initialized by Stutzer (1996) for pricing European option. The key of this method is to transform an empirical distribution of the historical underlying return into a posterior distribution via the maximum entropy principle by imposing the martingale constraint. This posterior distribution, thanking to the martingale constraint, can be treated as an appropriate risk-neutral distribution (RND) of the underlying return. With such risk-neutral measure, a European option can be easily priced by taking the discounted expectation of its

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payoffs at maturity. In canonical valuation, when achieving the risk-neutral measure, it relies more on the information contained in market prices and less on normative assumptions for the underlying price process, and for this reason the approach is also called model-free or nonparametric.

The canonical valuation is quite appealing for its nonparametric property, but a problem arises if the martingale constraint does not sufficiently restrict the feasible set of measures to enable the obtained pricing measure to be close enough to the correct martingale measure (Stutzer, 1996). To ease this problem, Gray, Edwards, and Kalotay (2007) use a European option traded on the prior day as an additional constraint forcing it to be correctly priced, so as to achieve a better pricing measure more tightly around the correct measure.

For the extension to this canonical valuation for pricing American options, all the existing entropy-based works combine the idea of Stutzer's valuation with the least-squares Monte Carlo algorithm (LSM, Longstaff & Schwartz, 2001). Within the Stutzer's framework, Alcock and Carmichael (2008) directly simulate the future paths of the underlying asset price based on a series of historical prices, and adopt a weighted LSM algorithm to determine the optimal exercise strategy. Liu (2010) contrarily obtains the risk-neutral pricing measure with daily gross returns and then generates the underlying paths (hence risk-neutral paths) with such measure prior to applying LSM to value the American option. These two approaches, however, use only the sole martingale constraint and ignore much valuable information (such as volatility information) contained in the option prices. To utilize more price-sensitive information, Alcock and Auerswald (2010) (abbreviated as AA10 hereafter) add on a certain 'European' call option price as a second constraint, suggested by Gray et al. (2007), to extend the previous work of Alcock and Carmichael (2008). The reported results show that this extended method for pricing American options yields substantial improvements compared with that of Alcock and Carmichael (2008). Liu and Guo (2014) (abbreviated as LG14 hereafter) instead impose an implied variance constraint[1] which might capture some of the market implied volatility information, into the framework of Liu (2010), so as to achieve an improved risk-neutral measure for pricing American options. The empirical test demonstrates that this method significantly outperforms that of Liu (2010) using a sample of American S&P 100 index put options.

Although all the entropy-based extended approaches above improve the pricing accuracy for American options by adding on a second constraint based on the canonical valuation, they fail to exploit more useful information from option data[2] which can accurately capture the shape of RND, such as the information of volatility smile and that of tail behaviour of distribution. When pricing an American option, AA10 method individually uses a specific 'European-style' call option with the same strike price and the same time to maturity, as the second constraint. Thus only the information included in such a single option is incorporated and consequently only the American option with the same characteristics as the single option could be accurately priced. In LG14 method, the implied volatility is calculated by forcing an option on a previous trading day to be priced correctly, as AA10 does, LG14 also utilizes the information contained in options individually and hence requires a large number of individual options to be used to get a proper volatility surface. In addition, both methods need to compute different RNDs for different options being priced and too much computational burden is required.

Can we aggregate more efficient information contained in the option market? and how can the information content be exploited and utilized for pricing American options? This paper explores this possibility by incorporating the risk-neutral moments (RNMs) as informative constraints into the entropy framework. First, the non-central RNMs of the underlying asset return are recovered using a set of American options based on the moment formulas we derive via a characteristic function.

[1] It is suggested by Britten-Jones and Neuberger (2000), where a model-free model is presented for estimating volatility. Henceforth a variety of model-free models were studied in many literatures, see, for example, Bakshi, Kapadia, and Madan (2003) for stock return characterizing, Kotzé, Labuschagne, Nair, and Padayachi (2013) for an arbitrage-free implied volatility surface constructing, and Chang, Jimenez-Martin, McAleer, and Amaral (2013) for VIX option trading.

[2] It is well known that option prices contain information about market participants' perceptions of the underlying distribution. See, for example, Jarrow and Rudd (1982), Aït-Sahalia and Lo (1998), and Jackwerth (1999) for a comprehensive review. Bates (1991), Grundy (1991), Day and Lewis (1992), Jackwerth and Rubinstein (1996), Melick and Thomas (1997), Bakshi and Madan (2000), Jiang and Tian (2005), and Diavatopoulos, Doran, Fodor, and Peterson (2012) for methodologies and application issues.