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Autoregresive conditional volatility, skewness and kurtosis

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Abstract

This paper proposes a GARCH-type model allowing for time-varying volatility, skewness and kurtosis. The model is estimated assuming a Gram–Charlier (GC) series expansion of the normal density function for the error term, which is easier to estimate than the non-central *t* distribution proposed by [Harvey, C. R. & Siddique, A. (1999). Autorregresive Conditional Skewness. *Journal of Financial and Quantitative Analysis* 34, 465–487). Moreover, this approach accounts for time-varying skewness and kurtosis while the approach by Harvey and Siddique [Harvey, C. R. & Siddique, A. (1999). Autorregresive Conditional Skewness. *Journal of Financial and Quantitative Analysis* 34, 465–487] only accounts for non-normal skewness. We apply this method to daily returns of a variety of stock indices and exchange rates. Our results indicate a significant presence of conditional skewness and kurtosis. It is also found that specifications allowing for time-varying skewness and kurtosis outperform specifications with constant third and fourth moments.

JEL classification: G12; G13; C13; C14

Keywords: Conditional volatility; Skewness and kurtosis; Gram–Charlier series expansion; Stock indices

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1. Introduction

There have been many papers studying the departures from normality of asset return distributions. It is well known that stock return distributions exhibit negative skewness and excess kurtosis (see, for example, Harvey & Siddique, 1999; Peiró, 1999; and Premaratne & Bera, 2001). Specifically, excess kurtosis (the fourth moment of the distribution) makes extreme observations more likely than in the normal case, which means that the market gives higher probability to extreme observations than in normal distribution. However, the presence of negative skewness (the third moment of the distribution) has the effect of accentuating the left-hand side of the distribution. That is, the market gives higher probability to decreases than increases in asset pricing.

These issues have been widely analyzed in option pricing literature. For example, as explained by Das and Sundaram (1999), the well known volatility smile and smirk effects are closely related to the presence of excess kurtosis and negative skewness in the underlying asset returns distribution.

The generalized autoregressive conditional heteroscedasticity models, introduced by Engle (1982) and Bollerslev (1986), allow for time-varying volatility (but not for time-varying skewness or kurtosis). Harvey and Siddique (1999) present a way to jointly estimate time-varying conditional variance and skewness under a non-central *t* distribution for the error term in the mean equation. Their methodology is applied to several series of stock index returns, and it is found that autoregressive conditional skewness is significant and that the inclusion of skewness affects the persistence in variance. It is important to point out that the paper by Harvey and Siddique (1999) allows for time-varying skewness but still assumes constant kurtosis.

Premaratne and Bera (2001) have suggested capturing asymmetry and excess kurtosis with the Pearson type IV distribution, which has three parameters that can be interpreted as volatility, skewness and kurtosis. This is an approximation to the non-central *t* distribution proposed by Pearson and Merrington (1958). However, these authors use time-varying conditional mean and variance, but maintain constant skewness and kurtosis over time. Similarly, Jondeau and Rockinger (2000) employ a conditional generalized Student's-*t* distribution to capture conditional skewness and kurtosis by imposing a time-varying structure for the two parameters, which control the probability mass in the assumed distribution[1]. However, these parameters do not follow a GARCH structure for either skewness or kurtosis.

The purpose of this research is to extend the work by Harvey and Siddique (1999) assuming a distribution for the error term in the mean equation that accounts for non-normal skewness and kurtosis. In particular, we jointly estimate time-varying volatility, skewness and kurtosis using a Gram–Charlier (GC) series expansion of the normal density function, along the lines suggested by Gallant and Tauchen (1989).

It is also worth noting that, apart from the fact that our approach accounts for time-varying kurtosis while the one by Harvey and Siddique (1999) does not, our likelihood

[1] This generalized Student's-*t* distribution is based on the work by Hansen (1994).