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On the ranking of bilateral bargaining opponents

Ross Cressman [a], Maria Gallego [b,*]

[a] *Department of Mathematics, Wilfrid Laurier University, Waterloo, Ontario N2L 3C5, Canada*
[b] *Department of Economics, Wilfrid Laurier University, Waterloo, Ontario N2L 3C5, Canada*

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ABSTRACT

We fix the status quo (Q) and one of the bilateral bargaining agents to examine how facing opponents with different single-peaked utility functions over a unidimensional space affects the Nash, Kalai–Smorodinsky and Perles–Maschler bargaining solutions. We find that when one opponent's utility is a concave transformation of the other's, the agent doing the ranking prefers the more risk averse, easier to satisfy, opponent. When opponents' utilities are translations of each other, we find that the bargainer whose ideal point is farthest from Q prefers an opponent whose ideal is closest to his own. For the agent closest to Q, the ranking of opponents depends on the absolute risk aversion (ARA) of the opponents' utility functions. Another intuitive result emerges when opponents' preferences exhibit increasing ARA: the ranking of solutions and opponents' ideal points coincide. However, under decreasing ARA, the agent closest to Q prefers the opponent whose ideal is farthest from her own. We also study rankings when one opponents' utility is a combination of a concave transformation and a right translation of the other's. For the concave/DARA and convex/IARA combinations, the effects on the solutions reinforce one another. In the concave/IARA and convex/DARA cases, the effect is ambiguous.

1. Introduction

In social choice environments, bilateral bargaining situations occur for example when voters delegate bargaining to between party negotiations in three-party legislatures, to governments in international negotiations or to central and sub-national authorities in intergovernmental negotiations; or when unions represent workers in wage negotiations. Though not concerned with

* Corresponding author. Tel.: +1 519 884 0710 x2635#; fax: +1 519 888 1015.
E-mail addresses: rcressman@wlu.ca (R. Cressman), mgallego@wlu.ca (M. Gallego).

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delegation and its incentive problems, we study situations where negotiators use their preferences to bargain over outcomes in a unidimensional space. Choosing a delegate requires ranking the agreements reached between different bargaining pairs.

A bilateral Nash bargaining problem (Nash, 1950) is defined by the set of feasible utility payoffs (S) for each agent including the disagreement outcome (d) that prevails if negotiations fail over a set of alternatives (X). There are several solutions[1] to the bargaining problem (S, d). We examine the Nash (NS), Kalai–Smorodinsky (KS) and Perles–Maschler (PM) solutions to our problem and assume that the disagreement outcome is outside the bargainers' control.[2]

We show that the presence of bargainers with different utility functions in a unidimensional space where the players' utilities and the status quo matter renders ranking agreements and consequently the choice of bargaining opponent more difficult. In order to rank the agreements reached between different bargaining pairs, we must understand how two components of the opponent's preferences determine her bargaining position: her ideal point and the degree of concavity of her utility function. These characteristics jointly determine the opponent's *toughness in negotiation*. When facing a choice between two opponents, an agent prefers the one who is less tough since the agreement with this opponent is then closer to the agent's ideal.

We examine situations where agents bargain for example over policies rather than monetary rewards. Our players have *single-peaked* preferences over a unidimensional space with ideals located anywhere in this space.[3]

In Austen-Smith and Banks (1988), voters delegate policy-making to a three-party legislature. Parties have single-peaked quadratic preferences with different ideal policies in the unidimensional space. In minority situations, each party, chosen in the order of vote shares, makes a proposal to the legislature. If no party's proposal is approved, all parties receive a zero payoff. In equilibrium, the bilateral agreement reached between parties follows the ranking of the parties' ideal policies. We show that the two rankings may not coincide when players have more general utility functions.

In Gallego and Scoones (2007), voters elect one of three parties to represent them in intergovernmental negotiations. Since the elected State formateur engages in bargaining over a unidimensional policy space with its Federal counterpart, voters must rank anticipated agreements. In equilibrium, the location of agreements depend on the identity and toughness of the formateurs. When parties have quadratic utilities, the ranking of agreements and parties' ideals coincides. However, if the centre and right parties are more risk averse than the left party, the two rankings do not coincide. We add insights to their findings in a broader framework and show that the counterintuitive result is possible when agents' entire utility function matters.

Let L and R represent the two bargainers. To rank agreements between different (L, R) pairs, we fix one agent and allow this agent to face opponents who differ in their ideals and/or in the degree of concavity of their utility functions (a property connected to the Arrow–Pratt coefficient absolute risk aversion, ARA; Peters, 1992b). To avoid repetition of cases that are reflections of those that we consider, assume that the status quo Q and L's ideal point are to the *left* of R's ideal. We assume complete information in a riskless environment to isolate the effect of the opponents' utilities on the solutions.

Milgrom and Shannon (1994) show that when the choice set and the parameters of the model [4] can be rank-ordered *and* players' preferences satisfy their single-crossing (SC) condition,[5] the solution to their optimization problem is monotonic in these parameters (i.e., the solution satisfies monotone

[1] For excellent discussions on the solutions to Nash's (1950) bargaining problem see Peters (1992a) and Thomson (1994).

[2] In social choice environments the disagreement outcome is usually determined by previous agreements reached by perhaps a different pair of agents. The disagreement outcome represents the bargainers' fallback position when negotiations fail and is determined by the status quo policy. We use these terms throughout the paper.

[3] Single-peaked utilities are commonly used in social choice and political economy models (see Austen-Smith and Banks, 1999 or Persson and Tabellini, 2000). In Alesina and Rosenthal (1996) and Baron and Ferejohn (1989) parties have single-peaked utilities.

[4] In our model, the choice set corresponds to the set of feasible agreements and the single parameter set is either the opponent's ideal point or their degree of risk aversion.

[5] Ashworth and Bueno de Mesquita (2006), Alesina and Rosenthal (1996), Austen-Smith and Banks (1988) and Gans and Smart (1996) use translated single-peaked utilities to illustrate the SC condition.