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Variance-constrained canonical least-squares Monte Carlo: An accurate method for pricing American options

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ABSTRACT

The pricing accuracy of the canonical least-squares Monte Carlo (CLM) method can be improved significantly by incorporating innovatively a variance constraint in the derivation of the canonical risk-neutral distribution. This new approach is called the variance-constrained CLM (vCLM) in the paper. Operationally, the forward variance is set to be the square of the volatility implied under vCLM by the option's market price from a previous trading day. For 16,249 American-style S&P 100 index puts, vCLM produced an average absolute pricing error of 5.94%, easily outperforming CLM, a competing nonparametric approach, and a GARCH-based benchmark.

1. Introduction

Canonical valuation, first proposed by Stutzer (1996), has undergone a revival in the recent works of Gray, Edwards, and Kalotay (2007), Alcock and Carmichael (2008), Liu (2010), and Liu and Guo (2013). Under the constraint of risk-neutrality, canonical valuation transforms an empirical probability measure of the historical underlying prices – via the maximum entropy principle (hence the term

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"canonical" by Stutzer) – into an equivalent martingale measure (Hull, 2009; Joshi, 2003; Klebaner, 1998). Under this new risk-neutral measure, a European option can be priced by simply taking the discounted expectation of its expiry payoffs. Canonical valuation is therefore quite appealing, since a change of measure to the risk-neutral distribution is achieved without having to assume an explicit stochastic process for the underlying asset. As a result, no parameters – such as volatility – need to be estimated; for this reason it also is called nonparametric.

Unfortunately yet understandably, the effect of volatility smile is not handled satisfactorily by canonical valuation, simply because the smile information is not available within the observed historical underlying prices. To solve this problem, Gray et al. (2007) used the market price of a prior-day traded option as an additional constraint to achieve better prices and superior hedging outcomes for European options, as compared to the results of the Black–Scholes (BS) with implied volatilities.

The adaptation of canonical valuation to American options pricing takes three different routes. A straightforward extension was proposed by Alcock and Carmichael (2008). They directly simulated the projected future paths of the underlying price with historical price ratios (just as is done with canonical valuation), and used weighted least-squares regressions to incorporate the early exercise features. In contrast, Liu (2010) suggested the canonical least-squares Monte Carlo (CLM) method. CLM obtains the equivalent martingale measure based on daily gross returns. This measure is then used to simulate risk-neutral paths for the underlying price before the least-squares Monte Carlo (LSM) algorithm (Longstaff & Schwartz, 2001) is applied to price American options. A third approach, proposed recently by Liu and Guo (2013), generates simulated stock paths exactly as CLM does. From the terminal values of these paths, an implied binomial tree (Rubinstein, 1994) is constructed instead and then used to price American options.

All three approaches fail to consider the effect of volatility smile for American options, just as canonical valuation does for European options. Trying to remedy this drawback, Alcock and Auerswald (2010) incorporated the approach of Gray et al. (2007), by utilizing a prior-day traded option price as a further constraint. To avoid infinite recursiveness with the introduction of the option price as a constraint, they chose to use certain European calls as market price proxies. Their reported pricing results for American options were comparable to those of the Black–Scholes with implied volatilities.

Can the market price of an American option be employed as a second constraint? This paper explores this possibility, while addressing the issue of volatility smile under the CLM method. As a proxy for its market price, the option's implied variance can be used as an additional constraint within the CLM framework. Extensive empirical testing with more than 16,000 American-style S&P 100 index puts shows that the variance-constrained CLM (vCLM) is much more accurate than CLM.

The rest of the paper is organized as follows. Section 2 proposes the idea of using market-implied variance (or equivalently volatility) as a second constraint. This turns out to be a natural extension of CLM and a better approach than the use of market price as a constraint, since it avoids the problem of infinite recursiveness mentioned earlier. In Section 3, the pricing errors for the American-style S&P 100 index puts computed by this extended CLM method are presented and analyzed. For comparison, three benchmarks, namely CLM, the Alcock and Auerswald (2010) approach, and an adaptation of the GARCH-based filtered historical simulation (Barone-Adesi, Engle, & Mancini, 2008), are employed. Then the paper concludes with a few comments. In the Appendix, true volatility smiles (not skews) and dynamic structures of the volatility surface for American index puts are documented.

2. Variance-constrained CLM method

2.1. Canonical risk-neutral distribution

As one of the three approaches mentioned in the Introduction, CLM extends canonical valuation of European options to price American options (Liu, 2010). To incorporate the early exercise features of an American option, CLM simulates underlying price paths by taking random samples from a set of K historical gross returns, $R_{\Delta t,k}$, $k = 1, 2, \ldots, K$. Here Δt can be a day, a week, or any other appropriate time interval. In practice, it is most convenient to work with daily intervals. Hereafter in this paper, the time interval Δt is assumed to be 1/365 years.