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Stationary distribution and periodic solutions for stochastic Holling–Leslie predator–prey systems[☆]

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HIGHLIGHTS

- Stochastic autonomous and periodic Holling–Leslie predator–prey systems are studied.
- Stationary distribution and ergodicity of autonomous system are obtained.
- Existence of the positive periodic solution of the periodic system is obtained.
- Simulations are given for stochastic Holling–Tanner and Holling-type IV system.

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ABSTRACT

The stochastic autonomous and periodic predator–prey systems with Holling and Leslie type functional response are investigated. For the autonomous system, we prove that there exists a unique stationary distribution, which is ergodic by constructing a suitable Lyapunov function under relatively small white noise. The result shows that, stationary distribution doesn't rely on the existence and the stability of the positive equilibrium in the undisturbed system. Furthermore, for the corresponding non-autonomous system, we show that there exists a positive periodic Markov process under relatively weaker condition. Finally, numerical simulations illustrate our theoretical results.

1. Introduction

To describe the predator–prey mite outbreak interactions on fruit trees in Washington State [1], Wollkind et al. [2] investigated the following equations based on the model proposed by May [3].

$$\begin{cases} \dfrac{dx}{dt} = x(t)(a_1 - b_1 x(t)) - y(t)p(x), \\ \dfrac{dy}{dt} = y(t)\left(a_2 - b_2 \dfrac{y(t)}{x(t)}\right), \end{cases} \tag{1}$$

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where $x(t),\ y(t)$ are the density function of the prey and the predator at time t, respectively. $a_i,\ b_i,\ (i = 1, 2)$ are all positive constants. The predator equation of (1) was first introduced by Leslie [4], based on the assumption that the carrying capacity of the predator in environment is proportional to the number of the prey. The growth of the predator is still of Logistic type with intrinsic growth rate a_2 and carrying capacity a_2x/b_2. The term $\frac{b_2}{a_2}\frac{y(t)}{x(t)}$ is the Leslie–Gower term, which measures the loss in the predator population due to rarity (per capita $y(t)/x(t)$) of its favorite food. And the Leslie–Gower formulation had been discussed by Leslie and Gower [5] and Pielou [6].

When $p(x)$ is of Holling type II, we have the following Holling–Tanner model:

$$\begin{cases} \dfrac{dx}{dt} = x(t)(a_1 - b_1x(t)) - \dfrac{ax(t)y(t)}{b + x(t)}, \\ \dfrac{dy}{dt} = y(t)\left(a_2 - b_2\dfrac{y(t)}{x(t)}\right), \end{cases} \tag{2}$$

where $a, b > 0$ denote capturing rate and half capturing saturation constant, respectively. Hsu and Huang [7] investigated the global stability of the positive equilibrium of the system (2) by applying the Dulac criterion and constructing the Lyapunov function. Saez and Gonzalez-Olivares [8] obtained the existence of a unique limit cycle of the system (2) in the first quadrant when the positive equilibrium becomes unstable. When $p(x)$ is of Holling-type IV, we have the following model:

$$\begin{cases} \dfrac{dx}{dt} = x(t)(a_1 - b_1x(t)) - \dfrac{ax(t)y(t)}{b + x^2(t)}, \\ \dfrac{dy}{dt} = y(t)\left(a_2 - b_2\dfrac{y(t)}{x(t)}\right). \end{cases} \tag{3}$$

Colling [9] showed that the bifurcation behaviors of the model (3) are sufficiently different from the dynamics of the system with Holling type I, II, III by numerical simulations. Li and Xiao [10] investigated the existence of the equilibria and Hopf bifurcation and Bogdanov–Takens of the system (3) when the parameters vary near the critical values, which implies that Holling-type IV functional response can result in the rich and complex dynamics.

However, in the real world, the natural growth of the populations is inevitably affected by the environmental white noise. Some authors introduce the random disturbance into the deterministic system to reveal the effect of the white noise, (see Refs. [11–16]). Simulated by the above work, we consider the system (1) with Holling functional response under the stochastic perturbation. Assume the intrinsic growth rates $a_1,\ a_2$ of the prey and the predator are disturbed with

$$a_1 \to a_1 + \alpha\dot{B}_1(t), \qquad a_2 \to a_2 + \beta\dot{B}_2(t),$$

where $B_1(t), B_2(t)$ are independent standard Brownian motions. $\alpha^2,\ \beta^2$ denote the intensities of the white noise. Then the deterministic systems (2) and (3) are disturbed into the following stochastic system:

$$\begin{cases} dx(t) = x(t)\left(a_1 - b_1x(t) - \dfrac{ay(t)}{b + x^n(t)}\right)dt + \alpha x(t)dB_1(t), \\ dy(t) = y(t)\left(a_2 - b_2\dfrac{y(t)}{x(t)}\right)dt + \beta y(t)dB_2(t), \end{cases} \tag{4}$$

where $n = 1, 2$. Ji, Jiang and Shi [11,12] considered a modified Leslie–Gower and Holling-type II predator–prey model and obtained that the system is persistent in time average and there exists a stationary distribution under the certain condition. In addition, Ji and Jiang [17] studied the existence of stationary distribution of the predator–prey system with Beddington–DeAngelis functional response by constructing the Lyapunov function. But in these work, the interaction terms of predator and prey are all bounded, and the constructions of Lyapunov functions all depend on the existence and the stability of the positive equilibrium. The aim of this paper is to prove the existence of a unique stationary distribution and ergodicity of the system (4) by constructing the suitable Lyapunov function, which does not depend on the existence and the stability of the positive equilibrium. What is more, under the condition that the interaction term of the system (4) is unbounded, we discuss the effect of stochastic disturbance, which appears to be completely new.

Moreover, due to the individual lifecycle and seasonal variation and so on, the birth rate, the death rate and the carrying capacity of the species and other parameters all exhibit cycle changes. However, as far as we know, only a few authors [18–21] studied the existence of periodic Markov process. In the article, we also consider the corresponding periodic system of (4):

$$\begin{cases} dx(t) = x(t)\left(a_1(t) - b_1(t)x(t) - \dfrac{a(t)y(t)}{b(t) + x^n(t)}\right)dt + \alpha(t)x(t)dB_1(t), \\ dy(t) = y(t)\left(a_2(t) - b_2(t)\dfrac{y(t)}{x(t)}\right)dt + \beta(t)y(t)dB_2(t), \end{cases} \tag{5}$$

where $a_i(t),\ b_i(t)\ (i = 1, 2),\ a(t),\ b(t),\ \alpha(t),\ \beta(t) > 0$ are all positive ω-periodic functions. Ding [22] studied the existence of the periodic solution for a semi-ratio-dependent predator–prey system without stochastic disturbance by using