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Dynamics of Brownian motors in deformable medium

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HIGHLIGHTS

- Directed transport of Brownian motors in deformable potential.
- Influence of the travelling wave speed on the dynamics of the system.
- The efficiency of generating the force is affected by the geometry.
- The travelling wave speed favours the transport in deformed systems.
- It is always advantageous to consider the whole range of the shape.

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ABSTRACT

The directed transport in a one-dimensional overdamped, Brownian motor subjected to a travelling wave potential with variable shape and exposed to an external bias is studied numerically. We focus our attention on the class of Remoissenet–Peyrard parametrized on-site potentials with slight modification, whose shape can be varied as a function of a parameter s, recovering the sine–Gordon shape as the special case. We demonstrate that in the presence of the travelling wave potential the observed dynamical properties of the Brownian motor which crucially depends on the travelling wave speed, the intensity of the noise and the external load is significantly influenced also by the geometry of the system. In particular, we notice that systems with sharp wells and broad barriers favour the transport under the influence of an applied load. The efficiency of transport of Brownian motors in deformable systems remains equal to 1 (in the absence of an applied load) up to a critical value of the travelling wave speed greater than that of the pure sine–Gordon shape.

1. Introduction

In the studies on Brownian motion, the perpetual irregular motions exhibited by small grains or colloidal particles of micrometric size maintained by the collisions with the molecules of the surrounding fluid can be probed. Examples of such Brownian particles are molecular motors such as kinesins whose importance is known in living biological cells and which have led to a great number of theoretical and experimental works in recent years [1–5]. Particularly, many experimental studies have been performed recently in the domain of living cells and showed the emergence of the anomalous diffusion;

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the relevant behaviour in these works is that although the mean displacement of the tracking particle is not linear, but random, its resulting motion is directed [4, and Refs therein]. The transport of Brownian particles along periodic structures in the apparent absence of any external driving forces, generally termed Brownian motor has been extensively studied [6–9]. Specifically, noise induced transport by Brownian motors or "rachets" has attracted the attention of an increasing number of researchers due to possible applications in many different contexts of physics, chemistry and biology [10–12].

The vast majority of works on Brownian motors is done in systems based on the standard sinusoidal potential and, concentrates on the behaviour and the selective control of the emerging directed transport as a function of parameters of the system such as temperature, energy barrier, or some other control variable. It is well known that under variation of some physical parameters such as temperature and pressure, certain physical systems may undergo changes which are either shape distortions, variation of crystalline structure or conformation changes. Thus, the standard sinusoidal potential used for modelling soft systems is interesting, but appears as a severe approximation because of the rigidity of its shape. In solid state physics deformable shape potential, which retrieves sine–Gordon shape as a special case has been employed widely and successfully to model the dynamics of systems in several realistic situations [13–17]. For example, recently, it has been revealed through the study of synchronization and information transmission in spatio-temporal networks that the final state of Frenkel–Kontorova oscillators was highly dependent on the initial conditions due to the shape of the system [18]. Similarly, it has been shown that the variations of the shape parameter affect significantly and not trivially the existence and the robustness of the velocity "quantization" phenomena [17]. Although the role of the shape parameter of the on-site potential is known in solid state physics, there exist only few information on its behaviour in soft condensed matter and biological systems [19–21]. Particularly in Refs. [20,21], the authors studied, the directed transport in asymmetric deformable systems and showed that there exists a value of the shape parameter at which the current takes its maximum. In those works, the only interest was the dependence of the current on the shape parameter of the system. In addition to the average drift velocity and/or current, each motor is characterized by the efficiency of converting the energy introduced by perturbations into useful work [2,22–24]. Moreover, the notion "travelling wave" introduced by Borromeo et al. [25] in their study of Brownian surfers where they showed its influence on the dynamics of an underdamped Brownian particle and used later by Li et al. [22] may also be considered.

In the present paper, the directed transport of Brownian particles in a travelling wave symmetric potential subjected to static bias is studied numerically. The generalization of the results [22] where the study of the influence of the travelling wave potential on the dynamical properties of Brownian particles for the shape parameter $s = 0$ is done by modelling the system, rather than with a standard sinusoid potential, but with the Remoissenet-Peyrard (RP) potential (with slight modification), whose shape can be varied continuously as a function of a shape parameter, and which acquires the sinusoidal shape as a special case [26,27]. Particular emphasis is laid on finding how the shape parameter of the system influences the directed transport in the case of symmetric travelling wave potential.

2. The model

Consider a one-dimensional Brownian particle with spatial position $x(t)$, γ the viscous friction constant which takes into account various sources of dissipation in the substrate (electronic excitations, phonons, etc.) and fluid (viscosity), subjected to an external static force or load F, plus a random thermal noise $\xi(t)$. In extremely small systems, particles dynamics and fluctuations occurring in biological and liquid environment are well described by the overdamped Langevin equation

$$\gamma \frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\mathrm{d}V\left(x - \mathrm{v}t, s\right)}{\mathrm{d}x} - F + \xi(t), \tag{1}$$

where the coupling between Brownian particles and the thermal bath is represented by $\xi(t)$, a standard Gaussian white noise of zero average and correlation $\langle \xi(t)\xi(t')\rangle = 2D\delta(t - t')$, with $D = k_B T/\gamma$ the noise intensity. Throughout this work, γ is set equal to one. The travelling wave potential $V(x - \mathrm{v}t, s)$ is assumed to be the RP potential with slight modification [26,27] written as

$$V\left(x - \mathrm{v}t, s\right) = U\left[\frac{(1+s)^2\left[1 - \cos\left(x - \mathrm{v}t\right)\right]}{1 + s^2 - 2s\cos\left(x - \mathrm{v}t\right)} - 1\right], \quad |s| < 1. \tag{2}$$

The quantity U and v are respectively the amplitude and the driving speed of the deformed travelling wave potential. In the absence of the driving speed ($\mathrm{v} = 0$), Fig. 1 represents the deformable substrate potential $V(x, s)$ for a few values of the shape parameter s. For $s = 0$, the potential $V(x, s)$ yields a sinusoidal shape, for $s < 0$, a shape of broad wells separated by narrow barriers, and for $s > 0$, a shape of deep narrow wells separated by broad gently sloping barriers (see Fig. 1).

It is well known that this stochastic process can be recast in terms of the probability density $p(x, t)$ which satisfies the Fokker–Planck equation [10]

$$\frac{\partial p(x,t)}{\partial t} = -\frac{\partial}{\partial x}\left[-\frac{\partial V\left(x - \mathrm{v}t, s\right)}{\partial x} - F - D\frac{\partial}{\partial x}\right]p(x,t). \tag{3}$$