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Linear combination of power-law functions for detecting multiscaling using detrended fluctuation analysis

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HIGHLIGHTS

- A linear combination of power-law functions for adjusting DFA data is proposed.
- Different values of the scaling exponents are estimated by nonlinear least-squares fitting.
- Examples of crude oil market and heart rate variability are discussed.
- Transition from anti-correlated to correlated behavior was observed.

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ABSTRACT

In many instances, the fluctuation function obtained from detrended fluctuation analysis (DFA) cannot be described by a uniform power-law function along scales. In fact, the manifestation of crossover scales may reflect the simultaneous action of different stochastic mechanisms displayed predominantly within certain scale ranges. This note proposes the use of a linear combination of power-law functions for adjusting DFA data. The idea is that each power-law function recast the dominance of certain stochastic mechanisms (e.g., the mean-reversion and long-term trends) at specific scale domains. Different values of the scaling exponents are numerically estimated by means of a nonlinear least-squares fitting of power-law functions. Examples of crude oil market and heart rate variability are discussed with some detail for illustrating the advantages of taking a linear combination of power-law functions for describing scaling behavior from DFA.

1. Introduction

The detrended fluctuation analysis (DFA) is a widely used approach for detecting serial auto-correlations in sequences. The method is meant for non-stationary sequences that can be also affected by long-term trends. Briefly, the DFA is carried out along the following steps [1]. For a sequence, $x(k),\ k = 1, \dots, N$, the corresponding integrated sequence is computed as $y(k) = \sum_{j=1}^{k} (x(j) - \langle x \rangle),\ k = 1, \dots, N$, where the mean value of the sequence $x(k)$ is given by $\langle x \rangle = \frac{1}{N}\sum_{j=1}^{N} x(j)$. In the next step, the integrated sequence $y(k)$ is divided into non-overlapping boxes of equal length $N_b = [N/M]$. In each box of length N_b, one fits $y(k)$ using a polynomial function $p_l(k)$ of order l, which represents the trend in that box. The integrated signal $y(k)$ is detrended by subtracting the local trend $p_l(k)$ in each box of length N_b and the root-mean-square fluctuation is calculated:

$$F_x(N_b) = \left(\frac{1}{N} \sum_{k=1}^{N} (y(k) - p_l(k))^2 \right)^{1/2}. \quad (1)$$

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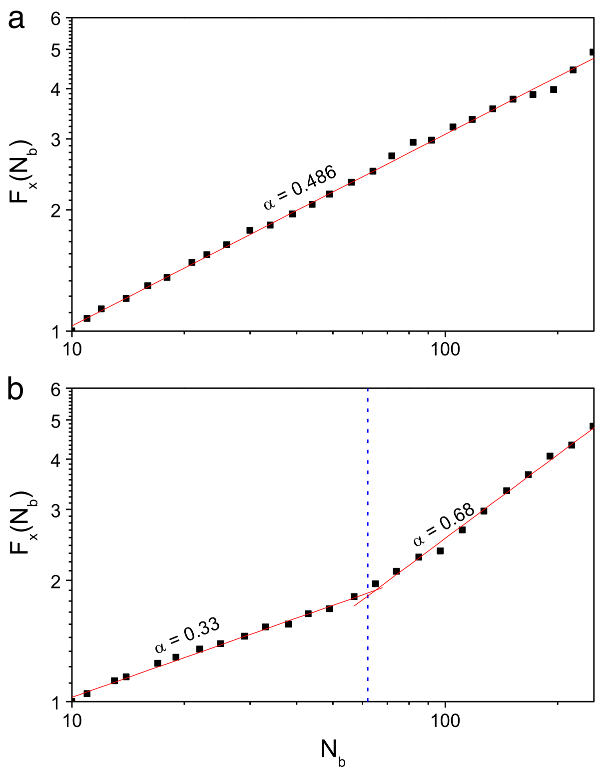

Fig. 1. (a) Scaling behavior of a random sequence. The estimated scaling exponent is near 0.5, reflecting the absence of long-term correlations. (b) Scaling behavior of price returns of WTI crude oil market. Two scaling patterns separated by a crossover scale can be observed.

The above computations are repeated for a broad range of scales (i.e., box sizes N_b) to provide a relation between the fluctuation function $F_x(N_b)$ and the box size N_b. To provide an accurate estimate of the fluctuation function $F_x(N_b)$, the smallest and largest box sizes are commonly taken of the order of $N_{b,\min} = 5$ and $N_{b,\max} = N/10$, respectively.

A reliable power-law relation between $F_x(N_b)$ and N_b is considered as an indication of scaling behavior of the original sequence $x(k)$:

$$F_x(N_b) = KN_b^{\alpha}. \tag{2}$$

The fluctuations with respect to the local trend are characterized by the scaling exponent α, which is a self-affine parameter representing the long-range correlations properties of the sequence $x(k)$. In this regard, there are no correlations (e.g., white noise) for $\alpha = 0.5$. On the other hand, the sequence is anti-correlated for $\alpha < 0.5$, and correlated for $\alpha > 0.5$.

In many instances, the fluctuation function $F_x(N_b)$ does not exhibit a uniform power-law behavior $F_x(N_b) \propto N_b^{\alpha}$ over the whole scale domain $[N_{b,\min}, N_{b,\max}]$. To illustrate this feature, consider a random (i.e., non-correlated) Gaussian sequence of length $N = 10^3$. Fig. 1(a) presents the fluctuation function $F_x(N_b)$ in log–log format for first-order polynomial detrending (i.e., $l = 1$) where a linear behavior related to the power-law $F_x(N_b) \propto N_b^{\alpha}$ can be observed. The power-law fitting, via linear least-squares adjustment of $\log(F_x(N_b))$ versus $\log(N_b)$, is also displayed, which provide a scaling exponent of 0.48 ± 0.02 over the whole scale range. Given the relatively short length of the random sequence, this is a good estimate of the scaling exponent for non-correlated patterns. Now, consider the sequence of WTI crude oil prices p_k for the period from 1986 to 1990, which corresponds to about 1060 observations. Take the sequence of daily returns $r(k) = p(k) - p(k-1)$ for analyzing the presence of correlations. Fig. 1(b) exhibits the fluctuation function $F_x(N_b)$ as function of the scale N_b in log–log format. In contrast to the case of the random sequence, the fluctuation function does not display a uniform power-law behavior. In fact, an apparent crossover scale is displayed at about $N_{b,cr} = 60$ business days.

Besides crude oil prices, non-uniform power-law behavior of the fluctuation function $F_x(N_b)$ appears in a diversity of instances, including wind-speed data [2], electro-seismic time series [3] vibrations in rotary machinery [4], heart rate variability [1,5], among many others. The proper detection of crossover scales can provide valuable insights in the transitions between different patterns, which can be used to monitor potential instabilities (e.g., in rotary machinery and heart rate dynamics) or to characterize mechanisms acting at short- and long-term horizons (e.g., crude oil markets). As further instances, a crossover scale at about 12–13 days was detected for geomagnetic field and seismicity at Etna volcano, which was linked to eruptive activity predictability [6]. Also, a crossover at about seven days was detected for high-frequency wind measurements in mountainous regions, which provides valuable insights in the long-term behavior of wind velocity aimed for eolic energy applications [7]. In general, the detection of crossover scales is in fact generally performed on the basis