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Entropy–complexity analysis in some globally-coupled systems

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HIGHLIGHTS

- The entropy–complexity plane enables to assess the chaotic or noisy nature of time series.
- This frame is applied to analyze the mean-field time series in a globally-coupled model.
- A Poincaré section analysis provides a natural sampling time for this analysis.
- Permutation entropy is shown to detect dynamical anomalies related to coherent structures.

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ABSTRACT

Globally-coupled N-body systems are well known to possess an intricate dynamics. When N is large, collective effects may drastically lower the effective dimension of the dynamics breaking the conditions on ergodicity necessary for the applicability of statistical mechanics. These problems are here illustrated and discussed through an entropy–complexity analysis of the repulsive Hamiltonian mean-field model. Using a Poincaré section of the mean-field time series provides a natural sampling time in the entropy–complexity treatment. This approach is shown to single-out the out-of-equilibrium dynamical features and to uncover a transition of the system dynamics from low-energy non-Boltzmann quasi-stationary states to high-energy stochastic-like behavior.

1. Introduction

In a seminal work [1,2], Wold established that any stationary process can be decomposed into the sum of a purely random and a purely deterministic process. Accompanying the modern construction of the theory of chaos has then emerged the quest for dynamical indicators that may discriminate between noise and deterministic chaos and may quantify the respective fractions of these. Given an experimental signal, one would typically like to know whether it is either of deterministic origin, being regular or chaotic, or of random nature, either partly or completely. In this line of research, a significant contribution came from the introduction of a measure of complexity for time series [3,4]. Recently, Rosso et al. introduced a representation space, the entropy–complexity causality plane [5], as a novel tool to analyze the chaotic and/or stochastic nature of dynamical systems. Testing this frame against well-known chaotic maps and stochastic processes, this work gave evidence that the entropy–complexity plane could offer a visual representation of the respective weights of the chaotic and stochastic (noise) components of time signals. Nevertheless the contours of the dynamical information devised under the term of 'complexity' (see e.g. the book [6] for an introduction to this concept) remain a subject of active research as well as the conditions of applicability of the entropy–complexity plane to continuous time signals instead of maps [7].

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In this study, it is proposed to consider the entropy–complexity frame as a new complementary tool to analyze the dynamics and transport properties of N-body systems. We have in mind long-range systems, and in particular mean-field systems, that are well known to exhibit both special equilibrium properties (e.g. with the possibility of ensemble inequivalence [8]) and relaxation properties [9–11] such as long-lived out-of-equilibrium states supported by collective waves or other quasi-stationary states. These are ergodicity-breaking features due to low collisionality impeding the relaxation towards Gibbs–Maxwell equilibrium and/or due to possibly insufficient intrinsic stochastic properties. Therefore these systems have a central role to play in the identification of the dynamical requirements for the validity and extensions of statistical mechanics [12–14]. Simultaneously, another objective of this study is also to test the entropy–complexity frame through its application to the HMF model, that is a more involved system than maps and a time-continuous system, contrarily to maps, but that is a more definite and well-controlled system than truly experimental signals, for which the entropy–complexity frame has started to be used with a vivid interest e.g. in plasma physics [15–17].

Because much of the information on the dynamics of N-body globally-coupled systems is usually contained in the time evolution of some low-dimensional subset of collective macroscopic variables, it is meaningful to focus on the characterization of the chaotic or stochastic properties of these few relevant collective variables. A prototypical example of such systems is the well-known Hamiltonian Mean Field (HMF) model [18] that will be introduced in Section 2. If the all-to-all particle coupling is repulsive, the HMF system was shown to exhibit some puzzling out-of-equilibrium dynamics in the low energy regime with the emergence of long-lived bicluster patterns whereas the equilibrium statistical mechanics predicts an homogeneous phase for all energies [19]. This transition will serve here to probe the entropy–complexity analysis. This will be introduced and discussed in Section 3. Then, a first dynamical signature of the transition between the very low-energy inhomogeneous states and upper-energy homogeneous ideal gas-like states will be exhibited in Section 4 through some continuous trace in the entropy–complexity plane. As a byproduct of this study, it will be emphasized that, in the large energy limit, the purely deterministic, and here almost non-chaotic,[1] HMF model produces a stochastic-like time behavior of the mean-field. This provides an illustration that time series having stochastic features can emanate from deterministic systems, which puts some limit on the possibilities of discriminating between the noisy or deterministic character of the governing dynamics. In Section 5, the issue of the influence of the sampling period in time-continuous systems on the estimation of the entropy and complexity indicators is addressed. Using a Poincaré section approach, by considering the time series of the relative maxima of the mean-field, is shown to enable to single-out in the entropy–complexity plane the low-energy regime where dynamical anomalies take place. A short conclusion evoking the potential applications and perspectives of this work is given in Section 6.

2. Dynamics of the globally-coupled repulsive HMF model

A paradigmatic conservative globally-coupled system is the so-called Hamiltonian Mean Field (HMF) model where N particles are moving on a circle being globally coupled by a cosine interaction with trajectories derived from

$$H(\mathbf{p},\mathbf{q}) = \sum_{i=1}^{N} \frac{p_i^2}{2} + \frac{c}{2N}\sum_{i,j=1}^{N}\left[1-\cos(q_i-q_j)\right]. \tag{1}$$

The interacting potential is of the same form as in the non-conservative Kuramoto model and both systems bear similarities. For instance, a puzzling $N^{1.7}$ scaling of the lifetimes of homogeneous quasistationary states was reported for the attractive HMF model [21] strangely resonating with the $N^{-1.69}$ scaling of the diffusion coefficient reported in chimera states of the Kuramoto model [22,23]. An explanation for this strange scaling was proposed in Ref. [24] in the HMF frame on the basis of a stochastic, diffusive approach.

Introducing the collective variable usually called the magnetization, using an analogy of the HMF potential with the X–Y spin model, defined by

$$\mathbf{M} = (M_x, M_y) = \left(\frac{1}{N}\sum_{i=1}^{N}\cos q_i, \frac{1}{N}\sum_{i=1}^{N}\sin q_i\right), \tag{2}$$

the equation of motion of any particle i may be written:

$$\frac{\mathrm{d}^2 q_i}{\mathrm{d}t^2} = -cM_x(t)\sin q_i + cM_y(t)\cos q_i. \tag{3}$$

For a negative coupling constant c, the equilibrium statistical mechanics of the HMF model predicts a vanishing canonical ensemble average of the modulus M of the mean-field. Moreover, the equivalence of the canonical and microcanonical ensembles has been proved for this system [9]. Therefore, from an equilibrium statistical mechanics point of view, the repulsive HMF model appears as trivial, and consequently, uninteresting. However, the numerical symplectic computations of its dynamics revealed some puzzling out-of-equilibrium features. Putting the constant c equal to -1, the minimal

[1] As long as the number of particles, N, in the HMF model is finite, the maximal Lyapunov exponent should be strictly positive, yet scaling as $N^{-1/3}$ [20].