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A multi-community homogeneous small-world network and its fundamental characteristics

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HIGHLIGHTS

- We propose a new concept to generate a small-world network having community structure.
- A generating algorithm is presented and its network parameters are explored.
- We discuss the fundamental characteristics of the proposed topology by using a spatial prisoner's dilemma game as template.

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ABSTRACT

We introduce a new small-world network – which we call the multi-community homogeneous-small-world network – that is divided into multiple communities that are relatively isolated, similar to sparsely connected islands. A generating algorithm is presented and its network parameters are explored. To elucidate the fundamental characteristics of the proposed topology, we adopt spatial prisoner's dilemma games as a template for discussion. Comparing with a conventional homogeneous small-world network, more enhanced network reciprocity is observed in games where a stag hunt-type dilemma is large. With intensive analysis, we find how this enhancement is brought about.

1. Introduction

We are ubiquitously surrounded by complex networks such as the Internet, our networks of acquaintances and friends, power grids, and logistics or supply networks. We can also observe many network systems in animal species such as ecosystems of cells or organisms, food chains, protein interactions, and relation in and out of animal groups.

For the last several decades, many scientists such as network physicists, biologists, and information engineers have been heavily concerned with the issue of complex networks, and considerable knowledge has been successfully compiled [1–6]. One related topic is the question of what features are important for explaining human social networks. Besides archetypical networks such as rings, lattices (in square, triangle, Kagome, and Bethe forms), and trees as well as regular random graphs (RRGs) and Erdös–Rényi (E–R) random graphs [2], which are convenient from a mathematical handling viewpoint, there have been two major characteristics commonly found in real human social networks. One is a "scale-free" (SF) relation observed in degree distribution and another is the so-called "small-world" (SW) characteristic. Just as an SF feature can be found in many natural systems, it also works behind human social relations, in which agents with a large number of acquaintances are few, whereas those who have a smaller number of acquaintances are numerous; in such systems, degree distribution obeys a power law. To model the SF features, Barabasi and Albert presented a generating algorithm based on the so-called preference attachment, which is used to produce the so-called Barabasi–Albert (B–A) SF networks [5–7], accepted

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by many people as the most standardized SF network. An SW network – an example of which, the ''six degrees of separation'' experimentally discussed by Milgram in 1967 [8], was originally posed by Watts [1] – can be featured with a relatively small average path length despite a huge number of vertices [5,9]. The most heavily applied SW networks are Watts–Strogatz [9] SW networks, in which one adds shortcut links obeying a shortcut probability (p_{short}) connecting two arbitrary vertices in either a ring or a lattice. Most previous SW networks including Watts–Strogatz SW networks have had heterogeneous degree distributions because adding shortcuts skews the original homogeneous topology provided by an underlying network that is either a ring or a lattice, implying that such networks are affected by not only SW effect or shortcut effects but also by the heterogeneity of the degree distribution. On this point, Santos et al. [10] proposed a new concept that they called homogeneous SW networks as opposed to heterogeneous SW ones, and confirmed, despite only small differences in network parameters such as average path length and cluster coefficient, that there is a non-negligible difference between the two SW networks when discussing the progression of epidemic-spreading phenomena as well as the function of network reciprocity in spatial prisoner's dilemma (SPD) games played on such network. Following this, several works [11,12] have studied SPD by comparing heterogeneous SW networks with homogeneous SW ones.

Another important property that we can find in real networks is their so-called community structure [4], by which the vertices cluster into several groups and the edges are more likely to connect to vertices within the same group, implying that network organization becomes modular. Since Girvan and Newman [13] first investigated community structure, many network scientists have proposed methods for detecting community in given networks [14], even though community is a qualitative concept with no widely accepted quantitative definition. Most of the artificially generated networks mentioned so far, such as rings, lattices, RRGs, E–R random graphs, B–A SFs, and conventional SWs, do not explicitly take community structure into account.

As noted above, the network science has been concerned on proposing new models to plausibly reproduce human social networks, because it has been expected to apply to various social physics aspects (consult several materials to capture an entire picture on the field [15–17]).

All of this background, more realistic complex networks representing human social relations must take account heterogeneity featured of SF as well as SW characteristic and community structure, which may propel to lead more practical works concerned on, for example, how and why a human social community allows to emerge mutual cooperation in its evolutionary process. Motivated by this, and so as to consolidate the first step, the present study aims to report a new topology and its generating model for homogeneous SW-embedded community structure, which we call the multi-community homogeneous-SW (Multi-Com Homo-SW) network. Here we see sparsely connected clique-like islands based on a lattice network, and replaced several original links with shortcuts to maintain homogeneity to introduce SW features.

The rest of the paper is organized as follows. Section 2 describes the generating algorithm of the Multi-Com Homo-SW network and discusses its network parameters; in Section 3, we apply SPD games to elucidate the features of Multi-Com Homo-SW networks as opposed to conventional SW networks in terms of network reciprocity; Section 4 draws conclusions.

2. Multi-community Homogeneous-Small-World Networks

2.1. Generating algorithm

Following the Watts–Strogatz SW model, let us start with homogeneous networks. In the following text, we presume a 2D lattice having degree k, for the sake of explanation. The total number of vertices (nodes) is defined as N. To generate an n-Community Homo-SW network, we prepare n independent lattices that, respectively, consist of N/n vertices. Those independent lattices are looped at their respective boundaries. As schematically shown in Fig. 1, each respective community, denoted community i, has two controlling parameters: $p_{short(i)}$ and $p_{same(i)}$. The former, $p_{short(i)}$, denotes shortcut probability. Hence, each community has $N \cdot k/n \cdot 0.5 \cdot p_{short(i)}$ shortcuts. The second parameter, $p_{same(i)}$, controls the destination vertex of a shortcut. In particular, with probability of $p_{same(i)}$, the destination vertex is randomly chosen from community i; otherwise (with probability of $1 - p_{same(i)}$), the shortcut is randomly connected to any arbitrary vertex in any arbitrary community.

Let us detail the procedure for introducing shortcuts by referring to Fig. 2. As mentioned above, preparing n lattices of size N/n, an arbitrary edge is randomly selected in any arbitrary community (let us presume community i). By random selection, the starting vertex, colored red, is determined. As shown in panel (a), the right for a random connection of the truncated edge is given to another vertex, colored yellow. A destination (from the yellow edge) for the random connection, colored green, should be selected from the community i with a probability of $p_{same(i)}$, and otherwise, form any arbitrary community. At the destination vertex (green), which is presumed to be located in community m in panel (b), one of the existing k edges, randomly selected, should be truncated. Likewise to the previous procedure, the right for random connection of the 2nd truncated edge is given to a vertex other than the destination vertex (the green edge). The next destination is then determined, which happens to be located in community i (highlighted by dotted gray box), as shown in panel (c). The above-noted procedure should be repeated until the total number of truncated edges becomes $N \cdot k/n \cdot 0.5 \cdot p_{short(i)}$. The last severed edge must be connected (from community j) to the initial point (in community i), colored red, to make sure that every vertex has k neighbors. The procedure until here should be repeated starting from an arbitrary edge selected in every other community to let the respective number of shortcut edges be $N \cdot k/n \cdot 0.5 \cdot p_{short(i)}$. Let alone, when dealing with community j obeying to the procedure above-mentioned, any other community is not allowed to have more shortcuts than