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Periodic solutions and stationary distribution of mutualism models in random environments

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HIGHLIGHTS

- The mutualism model perturbed by white noise admits a positive periodic solution.
- Existence, global attractivity of the boundary periodic solutions are discussed.
- The mutualism model perturbed by both white noise and color noise is discussed.

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ABSTRACT

This paper is concerned with mutualism models in random environments. For the periodic mutualism model disturbed by white noise, using Has'minskii theory of periodic solution, we show that this model admits a nontrivial positive periodic solution. Then sufficient conditions for the existence and global attractivity of the boundary periodic solutions are established. For the mutualism model disturbed by both white noise and telephone noise, sufficient conditions for positive recurrence and the existence of ergodic stationary distribution of the solution are established. Finally, examples are introduced to illustrate the results developed.

1. Introduction

In general, there are three main types of interaction in interacting population systems: predator–prey, competition and mutualism. Mutualism is an important interaction in nature. For example, rhinos and tick birds, flowering plants and their animal pollinators and ants and aphids are classic examples of such a relationship. Many mutualism models have been investigated extensively in the literature (see e.g. Refs. [1–5]). The famous mutualism model depicted by Lotka–Volterra equations can be described as follows

$$\begin{cases} \dot{x}(t) = x(t)\left(r_1 - a_1 x(t) + b_1 y(t)\right), \\ \dot{y}(t) = y(t)\left(r_2 - a_2 y(t) + b_2 x(t)\right). \end{cases}$$

There is an extensive literature concerned this model, for example, see Refs. [4–8].

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In 1976, May [9] initially proposed the following type of mutualism model

$$\begin{cases} \dot{x}(t) = x(t)\left(r_1 - \dfrac{b_1x(t)}{K_1 + \alpha_1 y(t)} - \varepsilon_1 x(t)\right), \\ \dot{y}(t) = y(t)\left(r_2 - \dfrac{b_2y(t)}{K_2 + \alpha_2 x(t)} - \varepsilon_2 y(t)\right), \end{cases} \tag{1}$$

where r_i, K_i, α_i and ε_i are all positive constants, $i = 1, 2$. System (1) has three trivial equilibrium points: $E_1 = (0, 0)$, $E_2 = (r_1/(\varepsilon_1 + b_1/K_1), 0)$, $E_3 = (0, r_2/(\varepsilon_2 + b_2/K_2))$ and a unique positive interior equilibrium point $E^* = (x^*, y^*)$ which is globally asymptotically stable, where x^* and y^* satisfy $r_1 - b_1x/(K_1 + \alpha_1 y) - \varepsilon_1 x = 0$ and $r_2 - b_2y/(K_2 + \alpha_2 x) - \varepsilon_2 y = 0$ [10].

However, in the real world, population systems are inevitably subjected to various environmental noises. There are two main types of environmental noises, one is white noise, the other is telephone noise. Suppose that intrinsic growth rates of model (1) are perturbed by white noise: $r_i \to r_i + \sigma_i \dot{B}_i(t)$, where $B_i(t)$ is a standard one-dimensional Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathbb{P})$. Then model (1) perturbed by white noise becomes

$$\begin{cases} \mathrm{d}x(t) = x(t)\left(r_1 - \dfrac{b_1x(t)}{K_1 + \alpha_1 y(t)} - \varepsilon_1 x(t)\right)\mathrm{d}t + \sigma_1 x(t)\mathrm{d}B_1(t), \\ \mathrm{d}y(t) = y(t)\left(r_2 - \dfrac{b_2y(t)}{K_2 + \alpha_2 x(t)} - \varepsilon_2 y(t)\right)\mathrm{d}t + \sigma_2 y(t)\mathrm{d}B_2(t), \end{cases} \tag{2}$$

where σ_i^2 is the intensity of the white noise, $i = 1, 2$. In Ref. [10], the authors studied the asymptotic behaviors of the solution to model (2) such as the existence and uniqueness of positive global solution, stochastic boundedness, uniform continuity, stochastic permanence and extinction. Besides white noise, model (2) may be disturbed by telephone noise which makes population systems switch from one regime to another. Therefore, stochastic mutualism model (2) under regime switching can be described by

$$\begin{cases} \mathrm{d}x(t) = x(t)\left(r_1(\xi(t)) - \dfrac{b_1(\xi(t))x(t)}{K_1(\xi(t)) + \alpha_1(\xi(t))y(t)} - \varepsilon_1(\xi(t))x(t)\right)\mathrm{d}t + \sigma_1(\xi(t))x(t)\mathrm{d}B_1(t), \\ \mathrm{d}y(t) = y(t)\left(r_2(\xi(t)) - \dfrac{b_2(\xi(t))y(t)}{K_2(\xi(t)) + \alpha_2(\xi(t))x(t)} - \varepsilon_2(\xi(t))y(t)\right)\mathrm{d}t + \sigma_2(\xi(t))y(t)\mathrm{d}B_2(t), \end{cases} \tag{3}$$

where $\xi(t)$ is a continuous time Markov chain with values in finite state space $S = \{1, 2, \ldots, N\}$, which depicts the telephone noise. One of our aims is to give sufficient conditions for the existence of a unique ergodic stationary distribution of the solution to system (3). The method we adopt is the ergodicity theory of stochastic models with regime switching in Zhu [11]. Using this method, Liu et al. [12], Settati et al. [13] and Zu et al. [14] have studied stationary distribution of stochastic Lotka–Volterra mutualism and predator–prey models.

It is well known that ecological environments (seasonal effects, food supplies, harvesting, etc.) change significantly through the year. So it is reasonable to introduce time-dependent parameters into the biological population models. Authors in paper [15] addressed that "most natural populations undergo seasonal fluctuation in abundance, which we attribute to seasonally varying factors (like food or energy) that limit population growth, i.e., density-dependent factors determining $K(t)$", where $K(t)$ is seasonal carrying capacity. In particular, the effect of periodic fluctuations has been addressed in several publications because, as mentioned by Vance and Coddington [16], "periodic time variation holds considerable promise as a means to explore time-varying ecological processes" [17]. And it is important to study the dynamics of population models with periodic environmental changes. Li and Xu [18] studied periodic solutions of stochastic delay differential equations. However, there are few papers focusing on the stochastic periodic solutions of population systems in random environments. Recently, using Has'minskii theory of periodic solution, Jiang et al. studied the existence of the periodic solutions to stochastic SIR model [19], Lotka–Volterra predator–prey model [20] and mutualism model [21]. Motivated by above analysis, in this paper, we consider the following stochastic two-species mutualism model

$$\begin{cases} \mathrm{d}x(t) = x(t)\left(r_1(t) - \dfrac{b_1(t)x(t)}{K_1(t) + \alpha_1(t)y(t)} - \varepsilon_1(t)x(t)\right)\mathrm{d}t + \sigma_1(t)x(t)\mathrm{d}B_1(t), \\ \mathrm{d}y(t) = y(t)\left(r_2(t) - \dfrac{b_2(t)y(t)}{K_2(t) + \alpha_2(t)x(t)} - \varepsilon_2(t)y(t)\right)\mathrm{d}t + \sigma_2(t)y(t)\mathrm{d}B_2(t), \end{cases} \tag{4}$$

where $r_i(t)$, $b_i(t)$, $K_i(t)$, $\alpha_i(t)$ and $\sigma_i(t)$ are all positive continuous T-periodic functions, $i = 1, 2$.

The remainder of the paper is organized as follows. In Section 2, we show that model (4) admits a nontrivial positive T-periodic solution and two boundary periodic solutions which are globally attractive. In Section 3, sufficient conditions for the existence of the ergodic stationary distribution, persistence in the mean and extinction are established for the model (3). Finally, numerical simulations are carried out to support the theoretical results.