Physica A 460 (2016) 38–43

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Mathematically modelling proportions of Japanese populations by industry

Yoshito Hirata

Institute of Industrial Science, The University of Tokyo, 4-6-1 Komaba, Meguro-ku, Tokyo 153-8505, Japan

HIGHLIGHTS

- The logistic equation extended to describe temporal changes of industrial structure.
- The extended model keeps variables non-negative whose sum equals to 100%.
- The extended model fits the Japanese historical data very well.
- The future prediction for industrial structural change is also provided.

ARTICLE INFO

Article history:
Received 18 December 2015
Received in revised form 19 March 2016
Available online 6 May 2016

Keywords:
Industrial populations
The logistic equation
Bifurcation analysis
Prediction

ABSTRACT

I propose a mathematical model for temporal changes of proportions for industrial sectors. I prove that the model keeps the proportions for the primary, the secondary, and the tertiary sectors between 0 and 100% and preserves their total as 100%. The model fits the Japanese historical data between 1950 and 2005 for the population proportions by industry very well. The model also predicts that the proportion for the secondary industry becomes negligible and becomes less than 1% at least around 2080.

1. Introduction

Industries, where employments are produced, are roughly divided into three sectors: primary industry, secondary industry, and tertiary industry. The primary industry includes agriculture, fishing, and forestry. The secondary industry includes manufacturing and mining. The tertiary industry is mainly the service industry.

Predicting the future proportions of populations by industry is necessary for a government to educate young generations so that demands and supplies of human resources match and fewer people become unemployed. Especially, the recent developments of robots and computers have been enabling us to make many jobs automatic, and/or do the same jobs with fewer people [1,2]. In such circumstances, protecting employments for people is getting important. We may be able to make the unemployment rate lower if people are educated and specialized in fields where robots and/or computers cannot replace human beings easily. However, as far as I noticed, there is no good model for predicting the future proportions of populations by industry except the one using correlation between two countries [3], which assumes that there is a country whose industrial structure will become similar to that of another country.

Here, I propose a dynamical mathematical model that describes temporal changes for the proportions of Japanese populations by industry [4]. I extend the famous logistic equation [5,6] in two ways because the logistic equation is a fundamental model of population dynamics: first, I introduce time dependent parameters; second, we couple the dynamics

E-mail address: yoshito@sat.t.u-tokyo.ac.jp.

http://dx.doi.org/10.1016/j.physa.2016.04.031
0378-4371/

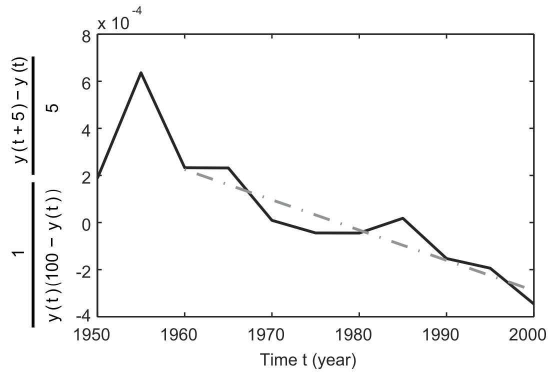

Fig. 1. Temporal changes for Eq. (2) for the proportion of population for the secondary industry in Japan. The black line shows the temporal changes. The grey dash-dotted line shows the linear fitting between 1960 and 2000, which shows a good fit (correlation coefficient: 0.93). If I obtain the correlation coefficient with the time axis between 1950 and 2000, the value becomes 0.82 (the p-value: 0.0018).

of population proportion for the secondary industry with those for the primary and tertiary industries in such a way that the proportions are constrained between 0 and 100% and the total for the proportions keeps being 100%. The proposed model can be fitted into the historical data well, and provide the prediction that the population for the secondary industry in Japan will become less than 1% at least around 2080 if the similar circumstances continue.

2. Dataset analysed

I use the historical dataset, available freely at Ref. [7], a site of Statistics Bureau, Ministry of Internal Affairs and Communications of Japan, for the proportions of populations by industry observed every 5 years between 1950 and 2005 to model and predict their temporal changes. Although this web site includes the proportions before 1950, I decided to use the part starting from 1950 to exclude the effects of the World War II.

3. Model

I extend the logistic equation [5,6] so that the mathematical model can describe the dynamics for the Japanese industrial populations. The logistic equation was originally introduced for describing the temporal changes for the population growth in human demography. Let y be the non-dimensionalized number of people. Then, in the logistic equation, the differential equation of y depending on time t can be written as

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \alpha_y y\,(100 - y)\,. \tag{1}$$

Here, I let y be the (non-dimensionalized) proportion for the people in the secondary industry in the asymptotic Japanese population, and thus I replace the constant inside of the brackets with 100 so that the variable y varies between 0 and 100. If the proportion for the population of the secondary industry follows Eq. (1), the following Eq. (2), obtained by dividing both sides of Eq. (1) by $y(100 - y)$, becomes a constant over time:

$$\alpha_y = \frac{1}{y\,(t)\,(100 - y\,(t))}\frac{\mathrm{d}y}{\mathrm{d}t}(t). \tag{2}$$

Here $\frac{\mathrm{d}y}{\mathrm{d}t}$ is the derivative of the proportion y with respect to t. To evaluate Eq. (2) in the empirical dataset, I use the difference of successive proportions instead of the derivative $\frac{\mathrm{d}y}{\mathrm{d}t}$. The results shown in Fig. 1 mean that the value of Eq. (2) has been decreasing at a constant rate from 1960 on.

Based on the results of Fig. 1, one can construct the following mathematical model for the proportion of population for the secondary industry:

$$\frac{\mathrm{d}y}{\mathrm{d}t} = \alpha_y(t) y\,(100 - y)\,, \tag{3}$$

$$\alpha_y(t) = a_y + b_y\,(t - 1950)\,, \tag{4}$$

where t corresponds to the time in years in the western calendar. If I assume that the primary and tertiary industries, denoted by x and z, respectively, also follow the same types of models and introduce interactions among these three industries, I can