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Analysis of the equilibrium trip cost without late arrival and the corresponding traffic properties using a car-following model

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HIGHLIGHTS

- A car-following model is utilized to explore each commuter's trip cost without late arrival.
- The analytical results are testified by the numerical tests.
- The traffic flow properties in the equilibrium state are studied.

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ABSTRACT

In this paper, we first apply a generalized car-following model to study the commuter trip cost without late arrival from an analytical perspective; and then use the full velocity difference (FVD) model to verify the analytical results and explore the corresponding traffic properties from a numerical perspective. Finally, we explore the evolutions of traffic flow on a road with an open boundary under three traffic situations (i.e., the number of commuters is low, moderate, and high) and find that the evolution of traffic flow is related to the number of commuters. The numerical results are qualitatively consistent with the analytical results and illustrate that car-following models can be used to study each commuter's trip cost without late arrival and that the car-following model can accurately quantify each commuter's trip cost.

1. Introduction

To date, peak-hour traffic congestion has become a great challenge to the sustainable development of modern society. For instance, the congestion cost in the U.S. has grown by approximately four-fold in the past 30 years [1]. Some transportation agencies are paying more attentions to vehicle emissions and energy consumption because they should measure traffic congestion as well as on-road mobile source emissions to be eligible for federal funding sources, e.g., the Congestion Mitigation and Air Quality Improvement program under the Moving Ahead for Progress in the 21st Century Act. Recently, emerging vehicle technologies (e.g., plug-in hybrid electric vehicle and battery electric vehicle) have offered a promising tool for emission reduction in urban areas. Policy makers are interested in understanding the benefits of the introduction of

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electric vehicle in terms of emission reduction and energy saving. However, the traditional macroscopic traffic flow models only allow a coarse assessment of the effects of traffic congestion on emission and energy consumption. In this paper, we will develop a model which is not only able to analyze the macro traffic flow pattern, e.g., equilibrium trip cost and departure time choice, but also can provide vehicle speed profiles at a high temporal resolution, which are critical for estimating vehicle emission and energy consumption.

In the literature, many theoretical models have been proposed to study traffic dynamics during rush hours; nevertheless, most are extensions of the basic bottleneck model [2]. Smith [3] and Daganzo [4] studied the existence and uniqueness of the solution to the basic bottleneck model [2]. The basic bottleneck model has been widely extended to study the commuter trip cost primarily because of its simplicity [5–16]. However, the Vickrey's bottleneck model made a basic assumption that a vertical queue representing congestion only occurs when the arrival rate of commuters is larger than the bottleneck capacity. Therefore, the basic bottleneck model and its extensions cannot be used to study the dynamics of rush-hour congestion that results from a queue at the bottleneck upstream. To conquer this limitation, Newell [17] used the first order model [18,19] to explore the morning commute problem where a fixed number of identical commuters must travel on a road of constant width. Recently, DePalma and Arnott [20] gave a detailed analysis of a special case of the Newell model [17], derived a closed-form solution for the system optimal (SO) problem and a quasi-analytical solution for the UE (user equilibrium) problem, and further discussed the economic properties of the two solutions. The methods [17,20] can quantitatively describe certain relationships among each commuter's trip cost, departure time and cumulative flow under the SO and UE principles, but they can neither provide an explicit relationship between each commuter's trip cost and departure time nor calculate each commuter's instantaneous speed, acceleration, and travel time. Hence, the above models lack the ability to accurately reproduce each commuter's individual trip cost from a microscopic perspective and cannot provide detailed information that is needed for accessing emission and energy consumption.

To explore this problem, Tang et al. [21] used a car-following model to study each commuter's trip cost without late arrival and concluded that each commuter's trip cost and the system's total trip cost depend on the departure time of commuters, but they assumed that each commuter's departure time is pre-determined, so the method [21] is not realistic in modeling the morning commute problem. The fixed departure pattern will prevent commuter from reducing his trip cost by changing his departure time, so the pre-determined departure pattern may not be sustainable as an equilibrium. In this paper, we apply a generalized car-following model to study each commuter's trip cost without late arrival and the corresponding traffic properties.

2. Model formulation

Assuming the total demand for a single origin–destination road is N. To represent each commuter's trip cost, we define the nth commuter's trip cost as a combination of travel time and early arrival penalty [20], i.e.,

$$T_n = \alpha\left(t_{n,\mathrm{a}} - t_{n,\mathrm{d}}\right) + \beta\left(t_{N,\mathrm{a}} - t_{n,\mathrm{a}}\right), \tag{1}$$

where T_n is the nth commuter's trip cost; α, β are respectively the per unit cost of travel time and early arrival time, and $\beta < \alpha$; $t_{n,\mathrm{d}}, t_{n,\mathrm{a}}$ are respectively the nth commuter's departure time and arrival time. All commuters have the same work start time, i.e., the arrival time of the last commuter, $t_{N,\mathrm{a}}$.

Before formulating the problem, we in this paper present the following four assumptions for the model:

(i) The road is a single-lane road, and its length is L.
(ii) There are N homogeneous commuters; their origin and destination are respectively the beginning and ending points of the road; each commuter is assumed to drive alone, so the commuter number and vehicle number can be used interchangeably.
(iii) When the time headway is less than a certain threshold, the arrival rate at the origin exceeds the capacity and commuters experience queuing at the origin. There is a minimum time headway at the origin and the minimum time headway is long enough. The assumption can guarantee that there is no waiting time for each commuter to enter the road at the origin.
(iv) When the nth commuter arrives at the destination, he automatically leaves the road, and his following vehicle becomes the leading vehicle.

Based on the above assumptions, we can categorize the nth commuter's motion behavior into the following two cases:

(a) When $t_{n,\mathrm{d}} \le t \le t_{n,\mathrm{a}}$, the nth commuter's vehicle operates on the road according to the car-following relationship as follows,

$$\begin{cases} a_n(t) = \begin{cases} f\left(v_n(t), \Delta x_n(t)\right), & \text{if } n = 1 \\ f\left(v_n(t), \Delta x_n(t), \Delta v_n(t), \ldots\right), & \text{if } n > 1 \end{cases} \\ v_n(t+\Delta t) = v_n(t) + a_n(t)\Delta t \\ x_n(t+\Delta t) = x_n(t) + v_n(t)\Delta t + 0.5 a_n(t)(\Delta t)^2, \end{cases} \tag{2}$$

where a_n, v_n, x_n are respectively the nth commuter's acceleration, speed, and position; $\Delta x_n, \Delta v_n$ are respectively the nth commuter's headway and relative speed; f is the stimulus function; Δt is the time-step length. Note: Eq. (2) cannot