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Stochastic permanence of an SIQS epidemic model with saturated incidence and independent random perturbations[☆]

Fengying Wei [a,b,*], Fangxiang Chen [a]

[a] College of Mathematics and Computer Science, Fuzhou University, Fuzhou 350116, Fujian, PR China

[b] Department of Mathematics and Statistics, University of Helsinki, Helsinki 00014, Finland

HIGHLIGHTS

- An SIQS epidemic model with independent random perturbations is formulated and studied.
- The unique solution is shown to remain in the positive cone with possibility 1.
- The study shows the stochastic permanence and ultimate boundedness.
- Numerical simulations illustrate the main theoretical results.

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ABSTRACT

This article discusses a stochastic SIQS epidemic model with saturated incidence. We assume that random perturbations always fluctuate at the endemic equilibrium. The existence of a global positive solution is obtained by constructing a suitable Lyapunov function. Under some suitable conditions, we derive the stochastic boundedness and stochastic permanence of the solutions of a stochastic SIQS model. Some numerical simulations are carried out to check our results.

1. Introduction

In recent years, many scholars have discussed the dynamics of the infectious diseases ranging from the deterministic models to the stochastic models. For example, Lahrouz et al. [1] studied the extinction and the stationary distribution of a stochastic SIRS epidemic model; Dalal et al. [2] showed the non-negative solutions and asymptotic stability of a stochastic AIDS model, later they [3] considered a stochastic model for internal HIV dynamics and analyzed the asymptotic behavior of the stochastic model; Jiang et al. [4] investigated the long time behavior of the DI SIR epidemic model with two kinds of stochastic perturbations; Yang et al. [5] discussed the ergodicity and the extinction of the SIR and SEIR epidemic models by using of Lyapunov functions; Zhang et al. [6] compared the asymptotic behaviors between the stochastic and the deterministic SIQS models with nonlinear incidence; Maroufy et al. [7] analyzed the positivity, stability and permanence of the SIRS epidemic model in an open population; Witbooi [8] proposed an SEIR epidemic model with independent stochastic perturbations.

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* Corresponding author.
E-mail addresses: weifengying@fzu.edu.cn, wei.fengying@helsinki.fi (F. Wei), n130320006@fzu.edu.cn (F. Chen).

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One of the most important factors is the incidence, which makes the epidemic diseases spread in different routes. In other words, the incidence plays an important role when the epidemic diseases are discussed in recent literatures [9,10]. Herewith, we would like to mention a work by Capasso [11] who studied the cholera epidemic spread in Bari and introduced a saturated incidence

$$g(I)S = \frac{\beta I}{1+\alpha I}S.$$

The incidence covered the behavioral change and crowding effect of the infective individuals and avoided the unboundedness of the incidence by choosing suitable parameters.

In this paper, we always assume that the following assumptions are valid: The community is divided into three compartments: the susceptible, the infective and the quarantined individuals, say S, I and Q, respectively. The random perturbations are dependent on the white noises, which are directly proportional to the distances of $S(t), I(t), Q(t)$ from the steady-state values of S^*, I^*, Q^*, respectively. Moreover, all parameters of the model are assumed to be positive for further discussion.

Let us formulate and consider a stochastic SIQS epidemic model with saturated incidence

$$\begin{cases} dS(t) = \left[A - \dfrac{\beta SI}{1+\alpha I} - dS + \gamma I + \rho Q\right]dt + \sigma_1(S-S^*)dB_1(t), \\ dI(t) = \left[\dfrac{\beta SI}{1+\alpha I} - (\gamma+\delta+d)I\right]dt + \sigma_2(I-I^*)dB_2(t), \\ dQ(t) = \left[\delta I - (\rho+d)Q\right]dt + \sigma_3(Q-Q^*)dB_3(t), \end{cases} \tag{1}$$

where A is a constant recruitment rate of compartment S corresponding to birth and immigration; d is the natural death rate; γ and ρ are the rates at which the individuals recover and return to compartment S from compartment I and compartment Q, respectively; δ is the rate at which the individuals leaving compartment I to compartment Q; B_i $(i = 1, 2, 3)$ are independent standard Brownian motions; σ_i $(i = 1, 2, 3)$ are the intensities of the white noises.

For the model (1), if we take the intensities of the white noises $\sigma_i = 0$ $(i = 1, 2, 3)$, then the following deterministic SIQS model is obtained,

$$\begin{cases} \dot S(t) = A - \dfrac{\beta SI}{1+\alpha I} - dS + \gamma I + \rho Q, \\ \dot I(t) = \dfrac{\beta SI}{1+\alpha I} - (\gamma+\delta+d)I, \\ \dot Q(t) = \delta I - (\rho+d)Q. \end{cases} \tag{2}$$

It is easy to check that the basic reproduction number of the model (2) is

$$R_0 = \frac{A\beta}{d(\gamma+\delta+d)}.$$

Further, the model (2) only has two equilibria: the infection-free equilibrium $E_0 = (\frac{A}{d}, 0, 0)$ and the endemic equilibrium $E^* = (S^*, I^*, Q^*)$ where the components S^*, I^* and Q^* are computed as

$$S^* = \frac{(\gamma+\delta+d)(1+\alpha I^*)}{\beta},$$
$$I^* = \frac{[A\beta - d(\gamma+\delta+d)](\rho+d)}{d(\rho+d)[\beta+\alpha(\gamma+\delta+d)]+\beta d\delta},$$
$$Q^* = \frac{\delta}{\rho+d}I^*.$$

We can prove that the infection-free equilibrium E_0 of the model (2) is globally asymptotically stable for $R_0 \le 1$. If $R_0 > 1$, the infection-free equilibrium E_0 of the model (2) is unstable and the endemic equilibrium E^* is globally asymptotically stable (see Ref. [12] with $\alpha_1 = \alpha_2 = 0$).

The organization of our paper is performed as follows. In Section 2, the notations and the definitions are presented for further discussion. In Section 3, we exhibit the existence and uniqueness of the global positive solution, stochastic boundedness and permanence by using the Lyapunov functions and Itô's formula. In Section 4, some numerical simulations are carried out to illustrate our results.