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Landau–Lifshitz–Gilbert equation with symmetric coefficients of the dissipative function II

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HIGHLIGHTS

- A solution of the modified Landau–Lifshitz–Gilbert equation is presented.
- An expression for the damping coefficient and the frequency of the oscillations is obtained.
- It is shown that for large values of H, the frequency is inversely proportional to H^3.

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ABSTRACT

In a previous study (Salazar and Perez Alcazar, 2015) we obtained the modified Landau–Lifshitz–Gilbert equation. The modification consisted in proposing a method to convert to symmetrical the kinetic coefficients of this equation. In the present study we find the solution to the proposed equation. This solution shows that the M_x and M_y components of the magnetization have damped oscillations. We find the expressions for the damping coefficient and the frequency of the oscillations and show their graphs. Finally, we compare these graphs with those that correspond to the frequency of oscillations for the magnetization in the Landau–Lifshitz–Gilbert equation.

1. Introduction

The common way to introduce a damping term in the classic equations of motion for a physical system is to consider the Lagrange formulation of the equations of motion and to add a term dependent on the velocity with which the dynamic variables of the system linked to the damping change [1]. This term is obtained from the temporal derivative of the dynamic variable of a quadratic function called a dissipative function.

For the case of a ferromagnet, in which the dynamic variables are the components of the magnetization $\boldsymbol{M}$, a dissipative term must be added in the equation of motion. For the magnetic case the dissipative force is given by [1]:

$$F = \frac{1}{2}\sum_{i,k} \eta_{ik}\,(H)\,\dot{M}_i\dot{M}_k. \tag{1}$$

It is clear that the coefficient $\eta_{ik}(H)$, called the kinetic coefficient, is not symmetric with respect to the subindices i, k, as it depends on the magnetic field. The Onsager theory states that in this case we must have [2]:

$$\eta_{ik}(H) = \eta_{ki}(-H). \tag{2}$$

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As any second degree tensor, $\eta_{ik}(H)$ can be subdivided into a symmetric and an antisymmetric part:

$$\eta_{ik}(H) = \eta_{ik}(H)_S + \eta_{ik}(H)_A \tag{3}$$

where the first term in the right is the symmetric component of the kinetic coefficient $\eta_{ik}(H)$ and the second term is its antisymmetric component. Only the symmetric part of this coefficient contributes in Eq. (1). Taking into account the even character of $\eta_{ik}(H)_S$ with respect to the magnetic field, we write the symmetric component in the following form:

$$\eta_{ik}(H)_S = \eta_{ik}^{(0)} + \varsigma_{iklm} H_l H_m \tag{4}$$

where ς_{iklm} is symmetric with respect to the indices i, k and l, m, but not with respect to the exchange of i, k by l, m.

The dissipative function takes the form:

$$F = \frac{1}{2} \sum_{i,k} \left\{ \eta_{ik}^{(0)} + \sum_{l,m} \varsigma_{iklm} H_l H_m \right\} \dot{M}_i \dot{M}_k. \tag{5}$$

Following the same procedure of Gilbert [3], we find that

$$\frac{\partial M_i}{\partial t} = \gamma \, \varepsilon_{ikl} \left\{ M_k H_l - M_k \left\{ \sum_{l,p} \left\{ \eta_{lp}^{(0)} + \sum_{r,s} \varsigma_{lprs} H_r H_s \right\} \frac{\partial M_p}{\partial t} \right\} \right\} \tag{6}$$

where γ is the gyromagnetic ratio. This is the new form of the Gilbert Equation.

We write Eq. (6) in the following vector form:

$$\frac{\partial M}{\partial t} = \gamma \, M \times \left\{ H - \left(\eta + \varsigma H^2 \right) \frac{\partial M}{\partial t} \right\}. \tag{7}$$

The goal of this study is to find the solution for this equation. In order to do that, we consider that the field H is directed in the positive direction of the z-axis.

2. Solution of the Landau–Lifshitz–Gilbert modified equation

To find the solution of the Landau–Lifshitz–Gilbert modified equation, Eq. (7) is written in the form

$$\frac{\partial M}{\partial t} = \gamma \, M \times H - \gamma \gamma_0 M \times \frac{\partial M}{\partial t} \tag{8}$$

where

$$\gamma_0 = \eta + \varsigma H^2. \tag{9}$$

Remembering that the field H is in the positive direction of the z axis, then Eq. (8) has the following components:

$$\frac{\partial M_x}{\partial t} = H\gamma M_y - \gamma \gamma_0 \left(M_y \frac{\partial M_z}{\partial t} - M_z \frac{\partial M_y}{\partial t} \right) \tag{10}$$

$$\frac{\partial M_y}{\partial t} = -H\gamma M_x - \gamma \gamma_0 \left(M_z \frac{\partial M_x}{\partial t} - M_x \frac{\partial M_z}{\partial t} \right) \tag{11}$$

$$\frac{\partial M_z}{\partial t} = -\gamma \gamma_0 \left(M_x \frac{\partial M_y}{\partial t} - M_y \frac{\partial M_x}{\partial t} \right). \tag{12}$$

Eq. (11) is multiplied by i and added to Eq. (10), and we obtain:

$$\frac{\partial M_T}{\partial t} = -\mathrm{i}\gamma H M_T - \mathrm{i}\gamma \gamma_0 M_z \frac{\partial M_T}{\partial t} + \mathrm{i}\gamma \gamma_0 \frac{\partial M_z}{\partial t} M_T \tag{13}$$

where we take

$$M_T = M_x + \mathrm{i} M_y. \tag{14}$$

Eq. (13) takes the following form:

$$\frac{\partial M_T}{\partial t} = \mathrm{i} \frac{\gamma \gamma_0 \frac{\partial M_z}{\partial t} - \gamma H}{1 + \mathrm{i}\gamma \gamma_0 M_z} M_T. \tag{15}$$

This equation is written in the following form:

$$\frac{\partial M_T}{\partial t} = \left\{ \frac{\left(\gamma \gamma_0 \frac{\partial M_z}{\partial t} - \gamma H \right) \gamma \gamma_0 M_z}{1 + (\gamma \gamma_0 M_z)^2} + \mathrm{i} \frac{\left(\gamma \gamma_0 \frac{\partial M_z}{\partial t} - \gamma H \right)}{1 + (\gamma \gamma_0 M_z)^2} \right\} M_T. \tag{16}$$