Physica A 453 (2016) 305–315

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Wave function for dissipative harmonically confined electrons in a time-dependent electric field

Meng-Yun Lai [a], Xiao-Yin Pan [a,*], Yu-Qi Li [b]

[a] Department of Physics, Ningbo University, Ningbo 315211, China

[b] Shanghai Key Laboratory of Trustworthy Computing, East China Normal University, Shanghai 200062, China

HIGHLIGHTS

- A many-body generalization of the Quantum Brownian motion model perturbed by a time-dependent homogeneous electric field is proposed.
- The analytical wave function of this model is obtained.
- The Harmonic Potential Theory is generalized to the quantum dissipative systems.

ARTICLE INFO

Article history:
Received 5 September 2015
Received in revised form 23 December 2015
Available online 24 February 2016

Keywords:
Dissipative quantum systems
Brownian motion
Harmonic Potential Theorem

ABSTRACT

We investigate the many-body wave function of a dissipative system of interacting particles confined by a harmonic potential and perturbed by a time-dependent spatially homogeneous electric field. Applying the method of Yu and Sun (1994), it is found that the wave function is comprised of a phase factor times the solution to the unperturbed time-dependent (TD) Schrödinger equation with the latter being translated by a time-dependent value that satisfies the classical damped driven equation of motion, plus an addition fluctuation term due to the Brownian motion. The wave function reduces to that of the Harmonic Potential Theorem (HPT) wave function in the absence of the dissipation. An example of application of the results derived is also given.

1. Introduction

In the past decades much attention has been paid to the physics of dissipative quantum systems [1–8]. Usually there are two main approaches for the dissipative quantum systems, the first one is so called system plus bath approach. Namely, put the system S into an environment, a bath B of N harmonic oscillators interacting with the system through certain coupling. The quantization is applied to the whole system $S + B$ which is conservative, then the dissipation in the system S being obtained by eliminating the variables of the bath R from its corresponding Heisenberg equation. An extensively studied model of the dissipative quantum system is the quantum Brownian motion (QBM) [9–19]. Applying the system plus bath approach to the QBM will provide both the friction and the fluctuation force.

The second approach is the use of an effective Hamiltonian, which yields the dissipation equation through its Heisenberg equation automatically. The classical model of quantum damped harmonic oscillator (QDHO) [20,21] is a typical example that results from the application of the effective Hamiltonian approach to the QBM while ignoring the effect of the Brownian

* Corresponding author.
E-mail address: panxiaoyin@nbu.edu.cn (X.-Y. Pan).

http://dx.doi.org/10.1016/j.physa.2016.02.001
0378-4371/

motion. The effective Hamiltonian corresponding to QDHO is the so called Caldirola [22] and Kanai [23] (CK) Hamiltonian (see Eq. (14)) but originally derived by Bateman [24]. This approach is very convenient to treat some dynamical problems of dissipative process for both classical and quantum cases. However, the CK Hamiltonian alone is not sufficient to the describe the quantum dissipative problem [20] since it only takes damping into account while no fluctuating force involved. Moreover, there appears to be various problems like the violation of the Heisenberg uncertainty principle associated with this Hamiltonian, we refer the reader to Refs. [20,21,25] for more details. This discrepancy can be avoided by interpreting the CK Hamiltonian as one that describes a harmonic oscillator having a time-dependent mass with exponential accretion $m(t) = me^{\eta t}$ instead of being subjected to a friction force [26–29]. On the other hand, by adding a stochastic force in the equation of motion which will take care of the fluctuation due to the environment [26,30–34] will also overcome the problem. Other approaches to solve this issue include considering a consistent transition between the CK formalism and a logarithmic nonlinear Schrödinger equation [25]. The connection between the CK Hamiltonian and the system plus bath approach has been shown by Yu and Sun (YS) [35,36], whereby they obtained an exact solution for the wave function of the system plus the bath.

In the literature, the model of QBM treated usually is a one-body problem and has been attacked by various authors using many different methods, such as the evolution of the density matrix is studied by using the master equation [37,38] within Lindblad theory [39,40] which is under the Markovian approximation. The exact master equation for QBM was derived by the authors of Refs. [16,18]. More recently, an exact method called quantum state diffusion [41] method was employed [42–47] to this problem. Moreover, the exact dynamics of a QBM in the presence of a time-dependent (TD) external force was studied by the authors of Ref. [48]. The studies of two-body QBM are rare and probably were first addressed by the authors of Ref. [49]. To our knowledge, a many-body generalization of QBM was first proposed by the authors of Ref. [19], but they only studied the case of two mutually coupled harmonic oscillators.

In this work, we investigate the many-body generalization of QBM perturbed by a TD spatially homogeneous electric field. We show that the wave function is comprised of a phase factor times the solution to the unperturbed time-dependent Schrödinger equation with the latter being translated by a TD value that satisfies the classical damped driven equation of motion, plus an addition fluctuation term due to the Brownian motion. In the absence of dissipation, the wave function derived reduces to that of the harmonic potential theorem (HPT) wave function [50] which has a similar structure. Thus we extend the HPT to the case in the presence of dissipation.

The rest of the paper is organized as follows. In Section 2, we show that the Hamiltonian can be decomposed into the center-of-mass (CM) and relative motion part, the external electric field will only affect the former. Then the effective Hamiltonian for the CM motion part is derived. Its wave function then is obtained in Section 3. In Section 4 an example of the application is given, then the concluding remarks are made in Section 5.

2. The effective Hamiltonian for the center-of-mass motion

For simplicity, we start with the one-dimensional case. Consider a system of N_s harmonic oscillators with arbitrary mutual interaction $u(q_i - q_j)$ and ω_0 the frequency. The Hamiltonian of the system is

$$\hat{H}_S = \sum_{i=1}^{N_s} \frac{\hat{p}_i^2}{2m} + \sum_{i=1}^{N_s} \frac{1}{2} m\omega_0^2 {q_i}^2 + \sum_{i\neq j}^{N_s} u(q_i - q_j). \tag{1}$$

Without lose of generality, we can assume the system is moved into an environment of a bath at $t = 0$. The bath B is comprised of N_B harmonic oscillators B_i with mass m_i, coordinates x_i and frequency ω_i. Then the Hamiltonian of the bath is

$$\hat{H}_B = \sum_{j}^{N_B} \left(\frac{\hat{p}_j^2}{2m_j} + \frac{1}{2} m_j \omega_j^2 x_j^2 \right). \tag{2}$$

The Hamiltonian of the composite system of S and B has the following form

$$\hat{H}_0 = \hat{H}_S + \hat{H}_I + \hat{H}_B, \tag{3}$$

where

$$\hat{H}_I = \left(\sum_{j}^{N_B} c_j x_j \right) \cdot \sum_{i}^{N_s} q_i, \tag{4}$$

is the interaction between the system and bath. It should be pointed out that this Hamiltonian $\hat{H}_0$ can also be derived from the Independent-oscillator model [5] as shown by YS in treating the exact dynamics of a quantum dissipative system in a constant external field [51]. A spatially homogeneous TD driving electric field $E(t) = f(t)/q$ with $q = -e$ the charge of an electron is turned on at time $t = t_0 > 0$. We assume $f(t) = 0$ for $t \leq t_0$. Mathematically $f(t)$ can be described by some smooth function times a step function, i.e. $f(t) = \Theta(t - t_0) f_e(t)$. Hence, without loss of generality we can always set