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Equilibrium pricing in an order book environment: Case study for a spin model

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HIGHLIGHTS

- Microscopic models for financial markets entail decision making and price formation.
- We embed the Bornholdt–Kaizoji–Fujiwara spin model into the order book dynamics.
- We conclude that the artificial assumption of a fundamental price can be abandoned.

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ABSTRACT

When modeling stock market dynamics, the price formation is often based on an equilibrium mechanism. In real stock exchanges, however, the price formation is governed by the order book. It is thus interesting to check if the resulting stylized facts of a model with equilibrium pricing change, remain the same or, more generally, are compatible with the order book environment. We tackle this issue in the framework of a case study by embedding the Bornholdt–Kaizoji–Fujiwara spin model into the order book dynamics. To this end, we use a recently developed agent based model that realistically incorporates the order book. We find realistic stylized facts. We conclude for the studied case that equilibrium pricing is not needed and that the corresponding assumption of a "fundamental" price may be abandoned.

1. Introduction

In models for financial markets, the traders' decision to buy or to sell is typically mapped to supply and demand, which are then balanced to determine the resulting price change, see Refs. [1–4]. This equilibrium pricing is a classical concept in the economics literature [5,6]. On the other hand, a real stock exchange uses a double auction order book. It provides a trading platform to every registered participant in which all offers to buy or sell are listed, ensuring that all traders have the same information. A price is quoted whenever a buy or sell orders match. Thus, the real price formation is quite different from the concept of equilibrium pricing. Here, we confront a model based on equilibrium pricing with the dynamics of a double auction order book. We wish to check the model results in view of the more realistic price formation.

Due to the rich variety of existing equilibrium pricing models, we have to restrict ourselves to case studies. In a previous work [7], we looked at a rather simple-minded decision making model that we analyzed with equilibrium pricing and, alternatively, in an order book environment. Here, we wish to address the more advanced Ising-type-of spin models which are known to properly capture several aspects of financial markets. We choose the Bornholdt–Kaizoji–Fujiwara model [8,9] as a particularly interesting representative. We sketch its salient features in Section 2. To apply order book dynamics to this

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model, we employ an agent-based model that fully accounts for the double auction order book as used in stock exchanges. It obviously makes sense to choose a "minimalistic" agent-based model free of additional features that might influence the resulting picture. Such an agent-based model was recently put forward and successfully tested in Ref. [10]. In Section 3, we briefly present its setup and adjust it to the Bornholdt–Kaizoji–Fujiwara model. This amounts to applying the decision making part of the Bornholdt–Kaizoji–Fujiwara model, but to then let the order book work. In particular, this lifts the constraints due to equilibrium pricing. We statistically analyze selected resulting quantities in Section 4. We summarize and conclude in Section 5.

2. Bornholdt–Kaizoji–Fujiwara model

Considering their paramount success for the study of phase transitions in statistical mechanics, it is not surprising that the application of spin models, in particular those of the Ising type, to financial markets has a long history, see Refs. [8,9,11–16]. Bornholdt [8] introduced an additional coupling constant to the ferromagnetic nearest neighbor interaction which then couples the individual spins to the total magnetization. This is motivated by two conflicting economic driving forces:

1. "Do what your neighbors do", by aligning your spin to your neighbors.
2. "Do what the minority does", by coupling to the magnetization.

As there is no quantifiable stock price in the original version of the model [8], Kaizoji, Bornholdt and Fujiwara [9] extended it accordingly by setting up a stock market with two groups of traders. We refer to this version as Bornholdt–Kaizoji–Fujiwara model.

There are n *interacting traders* i whose investment attitude is represented by one spin variable $S_i(t) = \pm 1,\ i = 1, \ldots, n$ each. The dynamics of the spins is governed by a heat bath that depends on a local Hamiltonian $h_i(t),\ i = 1, \ldots, n$ and determines a probability q such that

$$S_i(t+1) = +1 \quad \text{with } q = \frac{1}{1+\exp(-2\beta h_i(t))} \tag{1}$$

$$S_i(t+1) = -1 \quad \text{with } 1-q. \tag{2}$$

Here, $S_i(t) = 1$ ($S_i(t) = -1$) represents a positive (negative) investment attitude, meaning that trader i buys (sells) the stock. The traders' perception of the market is driven by two kinds of information. Locally, he is only influenced by the nearest interacting traders, but globally, it will also affect him whether or not he belongs to the majority group. This is measured by the absolute value $|M(t)|$ of the magnetization

$$M(t) = \frac{1}{n}\sum_{i=1}^{n} S_i(t). \tag{3}$$

To accumulate wealth, the trader has to be in the majority group and the majority has to expand over the next trading period. However, if $|M(t)|$ already has a large value, further increase is hampered. Traders in the majority group then tend to switch to the minority to avert a loss. On the other hand, a trader in the minority group tends to switch to the majority in the quest for profit. Altogether, the larger $|M(t)|$, the larger is the tendency for any group member to switch sides. The local Hamiltonian $h_i(t)$ entering Eq. (2) reads

$$h_i(t) = \sum_{j=1}^{n} J_{ij} S_j(t) - \alpha S_i(t)|M(t)|, \tag{4}$$

with interaction $J_{ij} = J$ for nearest neighbors i, j and with $J_{ij} = 0$ for all other pairs i, j. The global coupling constant α is positive, $\alpha > 0$. We notice that effects such as herding, trend following or various kinds of bubbles (see *e.g.* Refs. [17–19] and references therein) are not explicitly included in the model. If they occur in the simulations to be carried out, they are due to statistical fluctuations.

How is the stock price $p(t)$ determined in this model?—A number m of *fundamentalist traders* is introduced whose decisions are driven by supply and demand. They assume to have a reasonable knowledge of the fundamental value $p^*(t)$ of the stock price. If the price $p(t)$ falls below that threshold $p^*(t)$, the *fundamentalists* buy the stock, otherwise they sells it. The *fundamentalists*' excess demand is given by

$$x^F(t) = am\Big(\log p^*(t) - \log p(t)\Big) = am \log \frac{p^*(t)}{p(t)}, \tag{5}$$

where a is a parameter characterizing how strongly the fundamentalists react to the price difference between fundamental value and current price. On the other hand, the *interacting traders*' excess demand is governed by the total magnetization,

$$x^I(t) = bnM(t) \tag{6}$$

with b being the corresponding strength parameter.