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Analysis of car-following model with cascade compensation strategy

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HIGHLIGHTS

- Cascade compensation mechanism was introduced into car-following system.
- The stability of traffic flow system under different compensation parameters was discussed.
- The performance of the model was analyzed in time and frequency domain respectively.

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ABSTRACT

Cascade compensation mechanism was designed to improve the dynamical performance of traffic flow system. Two compensation methods were used to study unit step response in time domain and frequency characteristics with different parameters. The overshoot and phase margins are proportional to the compensation parameter in an underdamped condition. Through the comparison we choose the phase-lead compensation method as the main strategy in suppressing the traffic jam. The simulations were conducted under two boundary conditions to verify the validity of the compensator. The conclusion can be drawn that the stability of the system is strengthened with increased phase-lead compensation parameter. Moreover, the numerical simulation results are in good agreement with analytical results.

1. Introduction

Traffic flow theory has been developed for about more than sixty years since the classical car-following model was proposed by Pipes [1]. Many outstanding modifications have been achieved in the past decades. The famous car-following model was named optimal velocity model proposed by Bando in 1995 [2]. After this time Lenz [3] modified car-following model with a consideration of multi-anticipative effect. Nagatani [4] and Sawada [5] took the nearest interaction into account in improved the car-following model which made the model more realistically in describing the traffic behavior. Jiang [6] proposed full velocity difference model by considering the relative velocity between the current vehicle and the immediately one. Xue [7] analyzed Jiang's model with a nonlinear analysis method and derived the Burgers, Kdv and mKdv equations. Konishi [8,9] applied state space equation to describe the car-following model under open boundary condition. Decentralized delay-feedback control model and coupled map car-following model were investigated. Hasebe [10,11] discussed car-following model with forward-looking effects and backward-looking effects theoretically. An arbitrary

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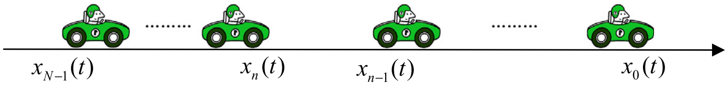

Fig. 1. Illustration of car-following model.

number of vehicles in the system that precede and follow were considered in the theoretical model. Ge [12] and Li [13] analyzed car-following model with forward-looking weighted headway and velocity difference in an intelligent transportation system. Zhao [14] improved full velocity difference model with acceleration difference and discussed it by using modern control theory. Tang [15–27] dedicated to improve the traffic flow model with a consideration of various factors in the real traffic system. Zhu [28,29] analytically and numerically investigated the behavior of the moving vehicles on a gradient and curved road respectively. Peng [30,31] improved the car-following with their unique viewpoint. Yu [32–35] verified the improved car-following model with the real field data collected with camera. These investigations are very helpful to the later study in improving the traffic flow theory. Recently traffic flow system was furthermore investigated by introducing some control strategy theory as proportional–differential effect [36] and speed feedback strategy [37] which can greatly improve the performance of the car-following system. Some compensation methods often are used to improve the performance of the control system. In this paper we will try to adopt linear cascade compensation methods to improve car-following system.

The remainders of this paper are organized as follows. In Section 2 the system was improved by introducing cascade compensation and analyzed by the use of the modern control theory. In Section 3, the improved model was verified with a unit step signal in time domain and frequency domain respectively. In Section 4, numerical simulations were conducted to verify the validity of the cascade compensator. In Section 5 the summary is given.

2. Model

Assume that all vehicles move on a single lane road without overtaking in an N-vehicle system (in Fig. 1). The leading vehicle is described as

$$x_0(t) = v_0(t) + x_0(0) \tag{1}$$

where $x_0(t)$ is the position of the leading vehicle, $v_0(t)$ is the speed of the leading vehicle, $x_0(0)$ is the initial position of the leading vehicle.

Based on Bando's optimal velocity model the motion equation of the system is given,

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = a \times [V(\Delta x_n(t)) - v_n(t)] \tag{2}$$

where, a is the sensitivity of the driver generally taken as $a = 0.85\ \mathrm{s}^{-1}$, $\frac{\mathrm{d}v_n(t)}{\mathrm{d}t}$ and $v_n(t)$ denote the acceleration and velocity of the nth vehicle at time t, $\Delta x_n(t) = x_{n-1}(t) - x_n(t)$ and its $x_n(t)$ denote the headway and position of the nth vehicle at time t, in which $n = 1, 2, \ldots, N$, N is the total number of the vehicles in the system, $V(\Delta x_n(t))$ is the optimal velocity function (OVF) of the nth vehicle [38] given as follows

$$V(\Delta x_n(t)) = \frac{v_{\max}}{2} \times (\tanh(0.13 \times (\Delta x_n(t) - s_c) - 1.57) + \tanh(0.13 \times s_c + 1.57)) \tag{3}$$

where, $v_{\max} = 15.82\ \mathrm{m/s}$, $s_c = 5\ \mathrm{m}$ which denotes the length of the vehicle.

In order to analyze the stability of the system we define the following steady state variables in the N-vehicle system

$$[v_n^*(t), \Delta x_n^*(t)]^T = [v_0, V^{-1}(v_0)]^T \tag{4}$$

where, $v_n^*(t)$ and $\Delta x_n^*(t)$ represent the steady state velocity variable and steady state headway variable of the nth vehicle at time t. The state implies that vehicles run orderly with speed v_0 and headway $V^{-1}(v_0)$.

Based on modern control theory, the vehicle system can be linearized around steady state as,

$$\begin{cases} \dot{X}_n(t) = AX_n(t) + BU_n(t) \\ Y_n(t) = CX_n(t) + DU_n(t) \end{cases} \tag{5}$$

where, $X_n(t)$ is system state variable vector. $U_n(t)$ is the input variable vector and $Y_n(t)$ is the output variable vector. These three variable vectors can fully describe the dynamics of traffic flow system.

$$\dot{X}_n(t) = \begin{bmatrix} \dfrac{\mathrm{d}(\tilde{v}_n(t))}{\mathrm{d}t} \\ \dfrac{\mathrm{d}(\Delta\tilde{x}_n(t))}{\mathrm{d}t} \end{bmatrix}, \qquad X_n(t) = \begin{bmatrix} \tilde{v}_n(t) \\ \Delta\tilde{x}_n(t) \end{bmatrix}, \qquad U_n(t) = \tilde{v}_{n-1}(t) \tag{6}$$