Physica A 449 (2016) 145–159

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Impulsive vaccination and dispersal on dynamics of an SIR epidemic model with restricting infected individuals boarding transports ☆

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HIGHLIGHTS

- An SIR epidemic model with restricting infected individuals boarding transports was investigated.
- Impulsive vaccination and dispersal are considered, which are more reasonable.
- Two thresholds are obtained for Infection-free of the system.

ARTICLE INFO

Article history:
Received 30 July 2015
Received in revised form 22 September 2015
Available online 3 December 2015

Keywords:
SIR model
Impulsive vaccination
Impulsive dispersal
Infection-free
Permanence

ABSTRACT

To understand the effect of impulsive vaccination and restricting infected individuals boarding transports on disease spread, we establish an SIR model with impulsive vaccination, impulsive dispersal and restricting infected individuals boarding transports. This SIR epidemic model for two regions, which are connected by transportation of non-infected individuals, portrays the evolvement of diseases. We prove that all solutions of the investigated system are uniformly ultimately bounded. We also prove that there exists globally asymptotically stable infection-free boundary periodic solution. The condition for permanence is discussed. It is concluded that the approach of impulsive vaccination and restricting infected individuals boarding transports provides reliable tactic basis for preventing disease spread.

1. Introduction

The work of Kermack and McKendick [1] was the fundamental study of epidemic models described by nonlinear differential equations. In this field, epidemic models have recently attracted much attention of mathematical epidemiologists and are perceived as significant [2–7]. Wang, Takeuchi and Liu [8] studied a multi-group SVEIR epidemic model with distributed delay and vaccination. Sun and Shi [9] considered a multigroup SEIR model with nonlinear incidence of infection and nonlinear removal functions between compartments. To understand the effect of transport-related infection on disease spread, Cui, Takeuchi and Saito [10] proposed the spreading disease with transport-related infection. Takeuchi, Liu and Cui [11] investigated the global dynamics of SIS models with transport-related infection, their conclusions implied that transport-related infection on disease can make the disease endemic even if all the isolated regions are disease free. Yan and Zou [12] considered two control variables representing the quarantine and isolation strategies for SARS epidemics, they gave a theoretical interpretation to the practical experiences that the early quarantine and isolation strategies are critically important

☆ Supported by National Natural Science Foundation of China (11361014, 10961008).

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http://dx.doi.org/10.1016/j.physa.2015.10.055
0378-4371/

to control the outbreaks of epidemic. Chowell and Castillo-Chavez [13] used the uncertainly and sensitivity analysis of the basic reproductive number R_0 to assess the role that the model parameters play in outbreak control. Quarantine and isolation measures have been widely used to control the spread of diseases such as yellow fever, smallpox, measles, ebola, pandemic influenza, diphtheria, plague, cholera, and, more recently, severe acute respiratory syndrome (SARS) [14–19]. Xie, et al. [20] simultaneously use two kinds of measures: expand the treatment ranges of suspected case and limit population flows freely to suppress the diffusion of SARS effectively. Gong et al. [21] showed that the SARS may fluctuate with import of SARS infectiousness from outside Beijing, weakness of quarantine, more social activities and so on.

Different types of vaccination policies and strategies combining pulse vaccination policy, treatment, pre-outbreak vaccination or isolation have already been introduced by many referees [22–31]. The pulse vaccination strategy (PVS) consists of repeated application of vaccine at discrete time with equal interval in a population in contrast to the traditional constant vaccination [22,23]. At each vaccination time a constant fraction of the susceptible population is vaccinated successfully. Since 1993, attempts have been made to develop mathematical theory to control infectious diseases using pulse vaccination [22]. Compared to the proportional vaccination models, the study of pulse vaccination models is in its infancy [23]. The control of childhood viral infections by pulse vaccination strategy is discussed by Nokes and Swinton [24,25]. Stone et al. [26] presented a theoretical examination of the pulse vaccination strategy in the SIR epidemic model. d'Onofrio [27] investigated the application of the pulse vaccination policy to eradicate infectious disease for SIR and SEIR epidemic models.

Theories of impulsive differential equations have been introduced into population dynamics lately [32–38]. Impulsive equations are found in almost every domain of applied science and have been studied in many investigation [38–41], they generally describe phenomena which are subject to steep or instantaneous changes. The theories of population dynamical system and its application have been achieved many good results. However, the oasis vegetation degradation combining with dynamical system has been considered very little. In this paper, we will investigate an impulsive dispersal on SIR model with restricting infected individuals boarding transports. We expect to obtain some dynamical properties of the investigated system. We also expect that impulsive dispersal will provides reliable tactic for controlling epidemic.

The organization of this paper is as follows. In Section 2, we introduce the model and background concepts. In Section 3, some important lemmas are presented. We give the globally asymptotically stable conditions of the infection-free boundary periodic solution of System (2.2), and the permanent condition of System (2.2). In Section 4, a brief discussion is given to conclude this work.

2. The model

Inspired by the above discussion, we establish an SIR model with impulsive vaccination, impulsive dispersal and restricting infected individuals boarding transports.

$$
\begin{cases}
\left.\begin{aligned}
\frac{dS_1(t)}{dt} &= \lambda_1 - d_1 S_1(t) - \frac{\beta_1 S_1(t) I_1(t)}{1+\alpha_1 I_1(t)},\\
\frac{dI_1(t)}{dt} &= \frac{\beta_1 S_1(t) I_1(t)}{1+\alpha_1 I_1(t)} - (r_1+d_1+b_1) I_1(t),\\
\frac{dR_1(t)}{dt} &= r_1 I_1(t) - d_1 R_1(t),\\
\frac{dS_2(t)}{dt} &= \lambda_2 - d_2 S_2(t) - \frac{\beta_2 S_2(t) I_2(t)}{1+\alpha_2 I_2(t)},\\
\frac{dI_2(t)}{dt} &= \frac{\beta_2 S_2(t) I_2(t)}{1+\alpha_2 I_2(t)} - (r_2+d_2+b_2) I_2(t),\\
\frac{dR_2(t)}{dt} &= r_2 I_2(t) - d_2 R_2(t),
\end{aligned}\right\} & t \neq (n+l)\tau,\ t \neq (n+1)\tau,\\
\left.\begin{aligned}
\triangle S_1(t) &= D(S_2(t) - S_1(t)),\\
\triangle I_1(t) &= 0,\\
\triangle R_1(t) &= D(R_2(t) - R_1(t)),\\
\triangle S_2(t) &= D(S_1(t) - S_2(t)),\\
\triangle I_2(t) &= 0,\\
\triangle R_2(t) &= D(R_1(t) - R_2(t)),
\end{aligned}\right\} & t = (n+l)\tau,\ n \in Z^+,\\
\left.\begin{aligned}
\triangle S_1(t) &= -\mu_1 S_1(t),\\
\triangle I_1(t) &= 0,\\
\triangle R_1(t) &= \mu_1 S_1(t),\\
\triangle S_2(t) &= -\mu_2 S_2(t),\\
\triangle I_2(t) &= 0,\\
\triangle R_2(t) &= \mu_2 S_2(t),
\end{aligned}\right\} & t = (n+1)\tau,\ n \in Z^+,
\end{cases}
\tag{2.1}
$$