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Hawkes-diffusion process and the conditional probability of defaults in the Eurozone

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HIGHLIGHTS

- Top-down analysis of the Eurozone debt crisis using a self-exciting jump model.
- We analyze the CDS term-structure data of 13 Eurozone countries.
- The contagion effect is significant during the period covering the Greek debt crisis.

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ABSTRACT

This study examines market information embedded in the European sovereign CDS (credit default swap) market by analyzing the sovereign CDSs of 13 Eurozone countries from January 1, 2008, to February 29, 2012, which includes the recent Eurozone debt crisis period. We design the conditional probability of defaults for the CDS prices based on the Hawkes-diffusion process and obtain the theoretical prices of CDS indexes. To estimate the model parameters, we calibrate the model prices to empirical prices obtained from individual sovereign CDS term structure data. The estimated parameters clearly explain both cross-sectional and time-series data. Our empirical results show that the probability of a huge loss event sharply increased during the Eurozone debt crisis, indicating a contagion effect. Even countries with strong and stable economies, such as Germany and France, suffered from the contagion effect. We also find that the probability of small events is sensitive to the state of the economy, spiking several times due to the global financial crisis and the Greek government debt crisis.

1. Introduction

Previous studies of financial economics and econophysics have attempted to find the determinants of sovereign default risks, presenting controversial arguments. One strand in the literature finds that country-specific factors mostly determine sovereign default risk [1,2]. Another strand claims that global financial variables explain a large part of the variation of sovereign risk [3–5]. Some studies investigate the role of non-credit risk and contagion risk [6,7]. Motivated by inconsistent findings in previous empirical studies, we aim to highlight a hidden risk factor, contagion risk, which is unobservable in financial markets, by applying a Hawkes-diffusion process to design sovereign default risk. The Hawkes-diffusion process is a variant of the self-exciting point process [8], which can generate the clustered jumps typical of historical default events. The

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self-exciting point process has been applied to a variety of fields, including financial economics, mathematics, and physics, to model the clustering of jumps [9–12]. We leverage the Hawkes-diffusion process to model the conditional probability of defaults in the Eurozone area and capture the hidden contagion risk factor.

The recent Eurozone debt crisis provides an ideal experiment to study the risk of default contagion. It is important for academics, practitioners, and policy makers to understand the contagion of sovereign risk for investment and risk management. Contagion risk within the Eurozone is concerning, especially regarding the potential exit of Greece from the Eurozone or "Grexit". During the Eurozone debt crisis initiated by the Greek debt crisis, even countries with relatively robust financial health, such as Germany, experienced a sharp increase in their sovereign CDS spreads (note that the level of sovereign CDS spreads is a good proxy of sovereign default risk). This implies that country-specific macro fundamentals alone do not seem to determine sovereign default risk. A default event (or the likelihood of an event) in Greece itself is another potential determinant of German sovereign risk, defined as "default contagion" in the literature. In other words, fear about a future event could increase premiums because investors want to be compensated for contagion risk.

While a structural model is typically used in analyses explicitly investigating the driving forces of default probability, the reduced-form approach used in this study is a statistical model under which the conditional probability of default (intensity) is exogenously determined and considered a risk factor without considering its economic meaning. Intensity is an exogenous factor in the existing intensity-based approaches, but we add a default risk factor to intensity so that default and intensity are endogenously determined. As a result, a sovereign default in the Eurozone area increases intensity (the conditional probability of another default). This model explicitly describes the contagion effect, or the hidden risk factor resolved at default.

Inspired by Errais et al. [13], Giesecke et al. [14], and Longstaff and Rajan [15], we employ a top-down approach to model the Eurozone default risk. The top-down approach focuses on modeling an overall portfolio loss process, while a bottom-up approach models the loss processes for each portfolio constituent to analyze the relationship between them.

Modeling an overall portfolio loss process has many advantages over modeling each constituent's process. First, the top-down approach can resolve our modeling and estimation problem. Also, the pricing formula of a credit portfolio with many constituents is tractable. Modeling a primitive top-down approach is a portfolio intensity process because a portfolio loss is a sufficient statistic. This allows us to use a univariate Hawkes process rather than a multi-variate Hawkes model to analyze contagion effects among the Eurozone countries. Our problem does not require us to specify each constituent country's intensity process one by one. Otherwise, modeling and estimating 13 countries' own risks and contagion risk between every pairs are virtually impossible. Papers using the top-down approach apply their models to bi-variate cases for simplicity and tractability [16,17].

The univariate Hawkes intensity in our model setting summarizes the default and contagion risk within the Eurozone as a whole, though it provides no information about the interaction (for example, lead/lag) among the countries. The feedback jump induces a sudden increase in instantaneous default (or loss) risk in the Eurozone. We do not intend to analyze each country's risk and the contagion mechanism within the Eurozone. Rather, we are mainly interested in measuring the contagion effect overall and determining the level of risk reflected in the term structure of CDS spreads.

To calibrate the model to the data, we use the term structures of sovereign CDS spreads. In addition, instead of averaging all individual spreads, we use the credit portfolio (index) construction method to extract the aggregate information. Section 2 describes the specifics in detail. Utilizing the Eurozone index spread gives us aggregate information on the recent Eurozone debt crisis, for which we decompose the estimated intensity into two events. One is a big event that causes a significant portion (estimated at 34.6%) of the sovereign portfolio to default, and another is a small event that leads to a small loss (estimated at 0.18%) on the sovereign bond portfolio when it arrives. The empirical result shows that the conditional probability of the small event is very sensitive to the variation of the macroeconomic state. For example, when the financial crisis became severe and Greece disclosed its sizable government debt, the intensity of the small event spiked several times. Interestingly, we find that the intensity of the big event remains low, even as the Global Financial Crisis progressed, but substantially increased when the Eurozone debt crisis spread. This offers evidence for the contagion effect within the Eurozone.

The remainder of this paper is organized as follows. In Section 2, we construct the model for CDS index spreads using a Hawkes-diffusion process. Section 3 provides empirical analysis, wherein we explain the data used, the estimation strategy, and the results of the analysis. Section 4 concludes the paper.

2. Methodology and model

2.1. Model development

In line with Longstaff and Rajan [15], we model the cumulative loss on a credit portfolio, $L(t)$, by a jump process, $N(t)$. Jumps induce instantaneous losses on the portfolio, and the loss size is determined by a random jump size, Y, realized as either γ_b with probability p or γ_s with probability $1-p$. We define a "big event" as when the random jump Y is realized by γ_b and a "small event" as γ_s, assuming $\gamma_b > \gamma_s$. From this setting, our model is summarized by

$$\mathrm{d}L(t)/(1-L(t)) = Y\,\mathrm{d}N(t). \tag{1}$$